Fuzzy Logic & Data Processing

Practice notes for Modern Method of Data Processing (CCOD)
in 2014

Akira Imada
Brest State Technical University, Belarus

(last modified on)

December 5, 2014

PART I

Fuzzy Controller

1 Vertual metro

PART II

Fuzzy Classifier

2 TS-fuzzy formula

We now assume n features and p rules:

Singleton Consequence

$$R_i\text{: IF } x_1 \text{ is } A_1^i \text{ AND } x_2 \text{ is } A_2^i \text{ AND } \cdots \text{ AND } x_n \text{ is } A_n^i \text{ THEN class is } g^i$$

Linear regression Consequence

$$\begin{gathered} R_i\text{: IF } x_1 \text{ is } A_1^i \text{ AND } x_2 \text{ is } A_2^i \text{ AND } \cdots \text{ AND } x_n \text{ is } A_n^i \\ \text{THEN } g_i = a_1^i x_1 + a_2^i x_2 + a_3^i x_3 + \cdots + a_n^i x_n + b^i \end{gathered}$$

Then, in both cases, class is estimated as follows:

$$\hat{y} = \frac{\displaystyle\sum_{k=1}^{p} M_k \cdot g^k}{\displaystyle\sum_{k=1}^{p} M_k}$$

where

$$M_k = \prod_{i=1}^{n} \mu_i^k(x_i)$$

where μ_i^k is the membership function of the attribute A_i^k.

2.1 Iris flower classification

2.1.1 Singleton consequance, triangle membership function

Apply the TS-fuzzy formula above to the iris flawer database, assumming the folloing $p = 3$ rules and membership functions[1].

[1]Taken frome H. Roubos et al. (2001) IEEE Transactions on Fuzzy Systems, Vol. 9, No. 4, pp. 516-524.

R_1: IF x_1 is short AND x_2 is long AND x_3 is short AND x_4 is short THEN y = 1.00.
R_2: IF x_1 is medium AND x_2 is small AND x_3 is medium AND x_4 is medium THEN y = 2.10.
R_3: IF x_1 is long AND x_2 is medium AND x_3 is long AND x_4 is long THEN y=2.95.

where each of the membership functions are adjusted as Figure 1 below.

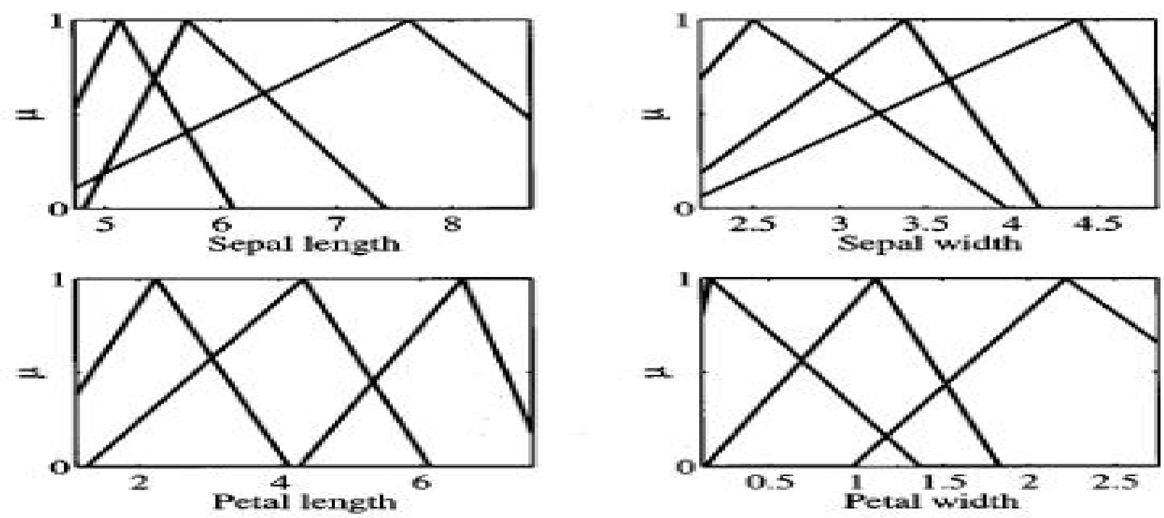


Figure 1: Triangle membership functions representing small, medium and large for x_1 (up left), x_2 (Up right), x_3 (bottom left) and x_4 (bottom left).

Apply TS-formula above and then estimated class is:

$$y = \begin{cases} 1 & \text{... if } \hat{y} < 1.5 \\ 2 & \text{... if } 1.5 \le \hat{y} < 2.5 \\ 3 & \text{... if } 2.5 \le \hat{y} \end{cases}$$

How many y out of 150 data are collect?

2.1.2 Linear regression consequance, Gaussian membership function

Apply the TS-fuzzy formula above to the iris flower database, assumming the folloing $p = 3$ rules and membership functions[2].

[2]Taken frome M.H. Kim et al. (2004) A novel appreoach to design of Takagi-Sugeno fuzzy classifier. Joint International Conference on Soft Computing and Intelligent Systems and International Symposium on Advanced Intelligent Systems.

$$\begin{bmatrix} a_1^1 & a_2^1 & a_3^1 \\ a_1^2 & a_2^2 & a_3^2 \\ a_1^3 & a_2^3 & a_3^3 \end{bmatrix} = \begin{bmatrix} -0.0000 & 0.0000 & -0.0001 \\ -0.1121 & -0.2234 & 0.0029 \\ -0.1020 & -0.0624 & 0.1276 \end{bmatrix}$$

and

$$\begin{bmatrix} b^1 \\ b^2 \\ b^3 \end{bmatrix} = \begin{bmatrix} 0.6667 \\ 1.7547 \\ 1.8412 \end{bmatrix}.$$

Gaussian membership function is defiend here as:

$$\mu(x) = \exp\{-\frac{(x-c)^2}{w}\}$$

where c and w represent center and width of distribution, respectively.

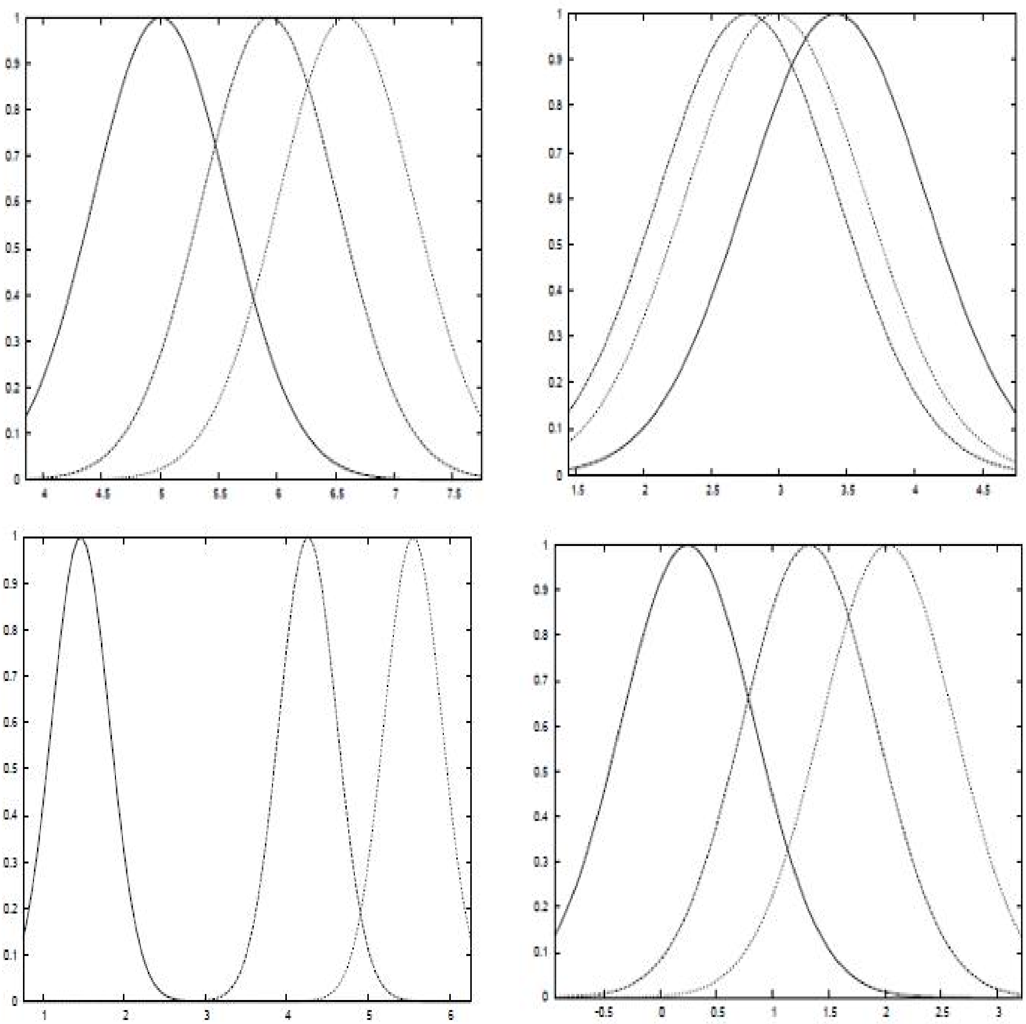


Figure 2: Gaussian membership functions representing small, medium and large for x_1 (up left), x_2 (Up right), x_3 (bottom left) and x_4 (bottom left).

2.2 Lenear Regression Consequence with Gaussian membership

Apply

R_i: If x_1 is A_1^i and x_2 is A_2^i and $\cdots$ and x_n is A_n^i then $y = a_1^1 x_1 + a_2^1 x_1 + \cdots + a_n^i x_n + b^i$.

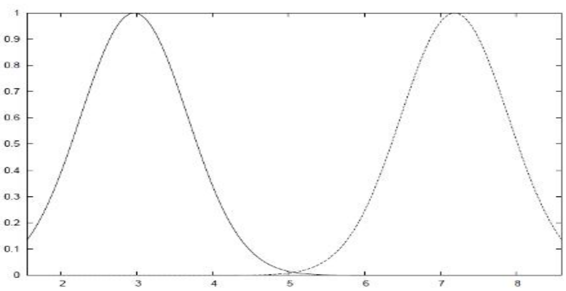
with a Gaussian membership function, that is,

$$\mu(x) = \exp\{-\frac{(x-c)^2}{w^2}\}$$

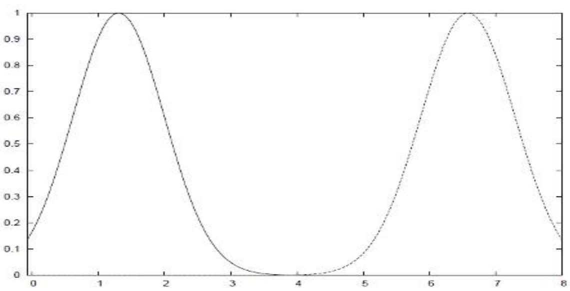
2.2.1 WBC data set

9 features and 2 classes

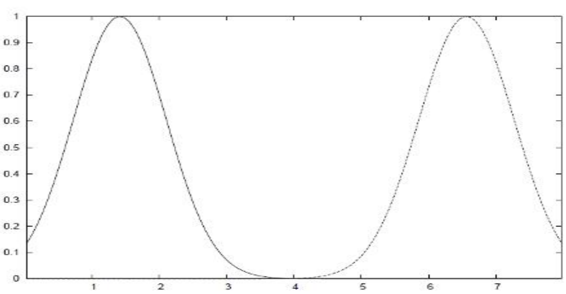
$$A = \begin{bmatrix} 0.0155 & 0.0125 \\ 0.0263 & 0.0021 \\ 0.0314 & 0.0030 \\ 0.0109 & 0.0051 \\ 0.0056 & -0.0012 \\ 0.0504 & 0.0052 \\ 0.0163 & 0.0096 \\ 0.0288 & 0.0015 \\ 0.0548 & 0.0028 \end{bmatrix}^T, \quad B = \begin{bmatrix} 0.6857 \\ 1.6990 \end{bmatrix}.$$



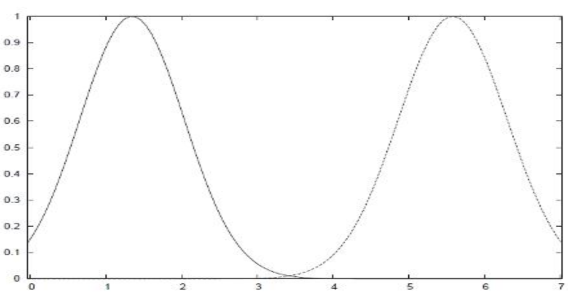
(a)



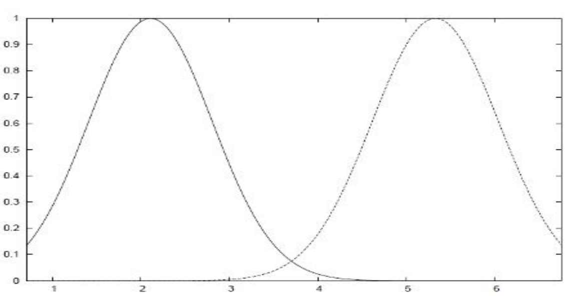
(b)



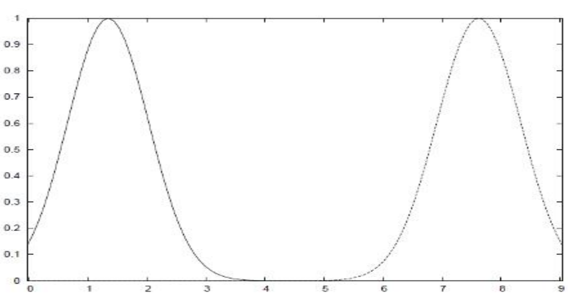
(c)



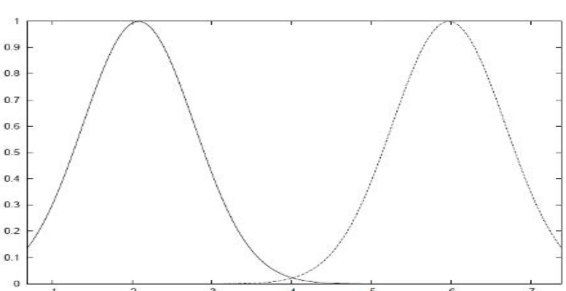
(d)



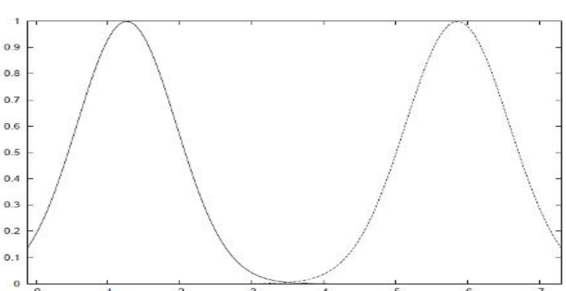
(e)



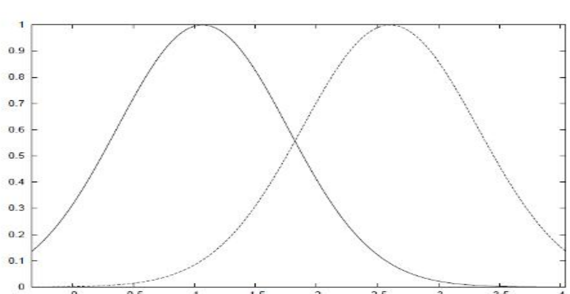
(f)



(g)



(h)



(i)

2.2.2 wine data set

13 features and 3 classes

3 Clustering by fuzzy relation

$$
R^{(0)} = \begin{bmatrix}
1 \\
0.2 & 1 \\
0 & 0.2 & 1 \\
0.4 & 0.6 & 0 & 1 \\
0.8 & 0.2 & 0 & 0 & 1 \\
0.4 & 0.2 & 0.6 & 0.4 & 0.2 & 1 \\
0.2 & 0.8 & 0.2 & 0.8 & 0.4 & 0.2 & 1 \\
0.8 & 0.8 & 0 & 0 & 0.6 & 0.4 & 0.4 & 1 \\
0.8 & 0.4 & 0.2 & 0 & 0.6 & 0.2 & 0.2 & 0.4 & 1 \\
0.2 & 0 & 0.6 & 0.2 & 0.2 & 0.8 & 0 & 0 & 0.4 & 1 \\
0.8 & 0 & 0 & 0 & 0.6 & 0.2 & 0.4 & 0 & 0.8 & 0.2 & 1 \\
0.2 & 0.8 & 0 & 0.2 & 0.2 & 0.2 & 0.8 & 0 & 0 & 0 & 0 & 1 \\
0.4 & 0.2 & 0.6 & 0 & 0.2 & 0.8 & 0.2 & 0 & 0.2 & 0.6 & 0 & 0 & 1 \\
0.8 & 0.2 & 0 & 0 & 0.8 & 0.4 & 0.2 & 0.6 & 0.8 & 0.4 & 0.8 & 0 & 0.4 & 1 \\
0 & 0 & 0 & 0 & 0.2 & 0.4 & 0.4 & 0 & 0.2 & 0.8 & 0 & 0 & 0.8 & 0.4 & 1
\end{bmatrix}.
$$

ckearoage

$$R^{(0)} = \begin{bmatrix} 1 & & & & & & & & & & & & & & \\ 0.2 & 1 & & & & & & & & & & & & & \\ 0 & 0.2 & 1 & & & & & & & & & & & & \\ 0.4 & 0.6 & 0 & 1 & & & & & & & & & & & \\ 0.8 & 0.2 & 0 & 0 & 1 & & & & & & & & & & \\ 0.4 & 0.2 & 0.6 & 0.4 & 0.2 & 1 & & & & & & & & & \\ 0.2 & 0.8 & 0.2 & 0.8 & 0.4 & 0.2 & 1 & & & & & & & & \\ 0.8 & 0.8 & 0 & 0 & 0.6 & 0.4 & 0.4 & 1 & & & & & & & \\ 0.8 & 0.4 & 0.2 & 0 & 0.6 & 0.2 & 0.2 & 0.4 & 1 & & & & & & \\ 0.2 & 0 & 0.6 & 0.2 & 0.2 & 0.8 & 0 & 0 & 0.4 & 1 & & & & & \\ 0.8 & 0 & 0 & 0 & 0.6 & 0.2 & 0.4 & 0 & 0.8 & 0.2 & 1 & & & & \\ 0.2 & 0.8 & 0 & 0.2 & 0.2 & 0.2 & 0.8 & 0 & 0 & 0 & 0 & 1 & & & \\ 0.4 & 0.2 & 0.6 & 0 & 0.2 & 0.8 & 0.2 & 0 & 0.2 & 0.6 & 0 & 0 & 1 & & \\ 0.8 & 0.2 & 0 & 0 & 0.8 & 0.4 & 0.2 & 0.6 & 0.8 & 0.4 & 0.8 & 0 & 0.4 & 1 & \\ 0 & 0 & 0 & 0 & 0.2 & 0.4 & 0.4 & 0 & 0.2 & 0.8 & 0 & 0 & 0.8 & 0.4 & 1 \end{bmatrix}.$$