A slide show of our practice note

Fuzzy Data Processing

Practice 2020 - online

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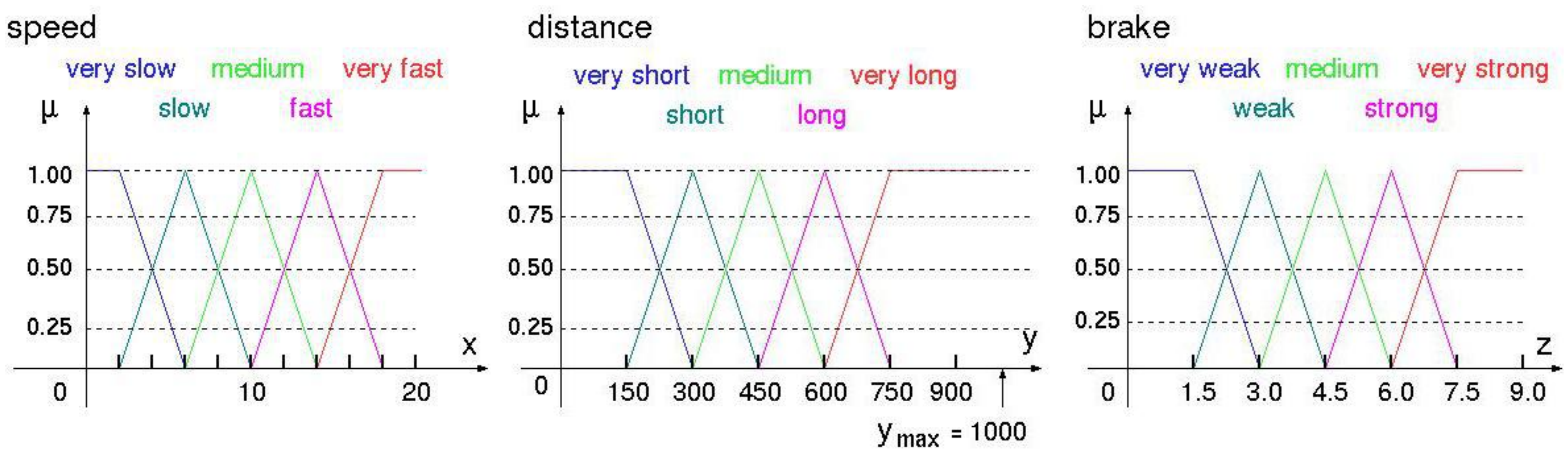
Last modified on 18 October 2020

I. How to control two virtual metro cars?

Assume 5 triangle membership functions for each of 3 categories



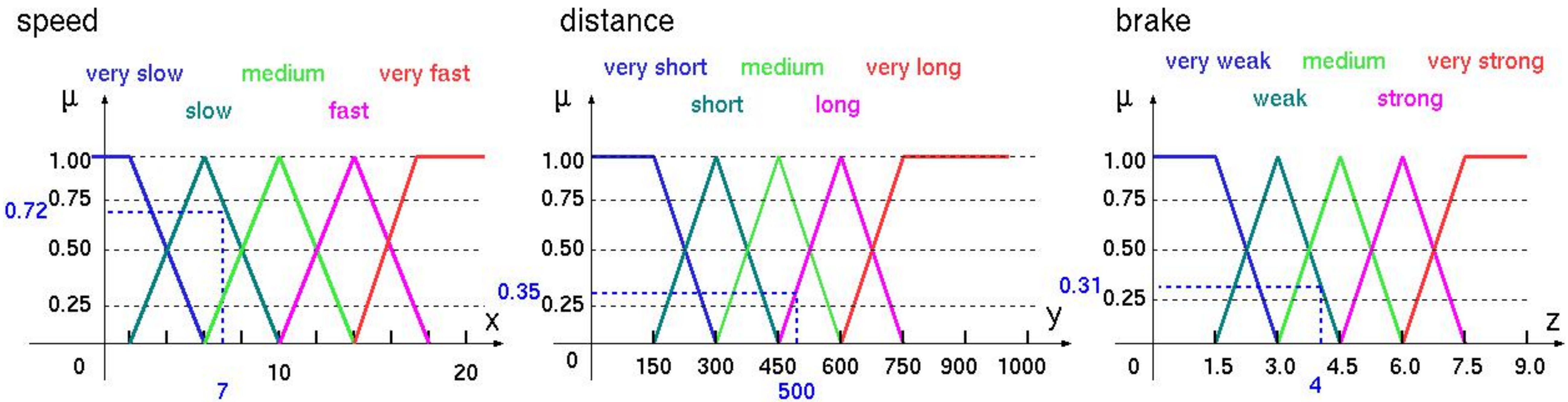
Membership of 3 specific values of speed, distance and brake

Under one rule

IF x = slow AND y = long THEN z = weak

Assume now x = 7, y = 500, z = 4

Then the membership value of this rule is ➜ min{0.72, 0.35} x 0.31 = 1.085



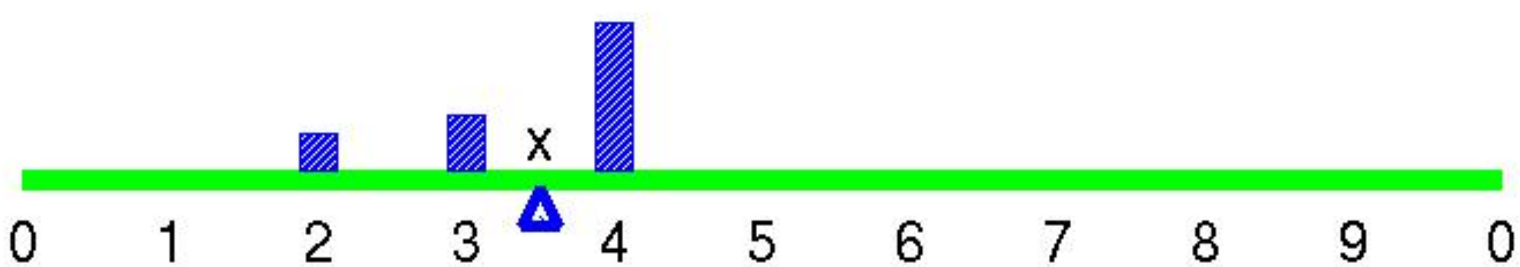
When Speed = 7 and distance = 500

under one rule

IF Speed is slow AND Distance is long THEN Brake is weak

brake	0	1	2	3	4	5	6	7	8	9	10
The membership value					1.085						
	⇑	⇑	⇑	⇑		⇑	⇑	⇑	⇑	⇑	⇑
	0.000	0.000	0.175	0.350		0.000	0.000	0.000	0.000	0.000	0.000

Fuzzify! => The best appropriate value of brake?



$0.175(x-2) + 0.350(x-3) + 1.085(x-4) = 0 \quad \Rightarrow \quad 1.610x = 5.740 \quad \Rightarrow \quad x = 3.57$

Membership of 3 specific values of speed, distance and brake

Under two rules

IF x = slow AND y = long THEN z = weak

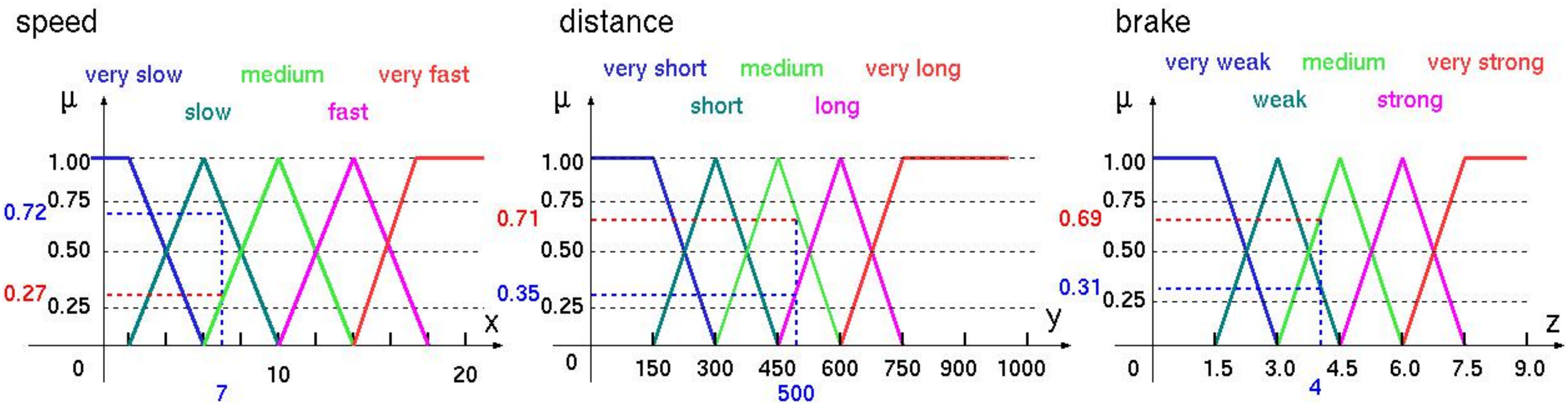
OR

IF x = medium AND y = medium THEN z = medium

Assume now x = 7, y = 500, z = 4

Then the membership value of these two rules is

max{min(0.72, 0.35)x0.31, min(0.27, 0.71)x0.69} = max{0.1085, 0.1823} = 0.1823



When Speed = 7 and distance = 500

under two rules

IF Speed is slow AND Distance is long THEN Brake is weak

IF Speed is medium AND Distance is medium THEN Brake is medium

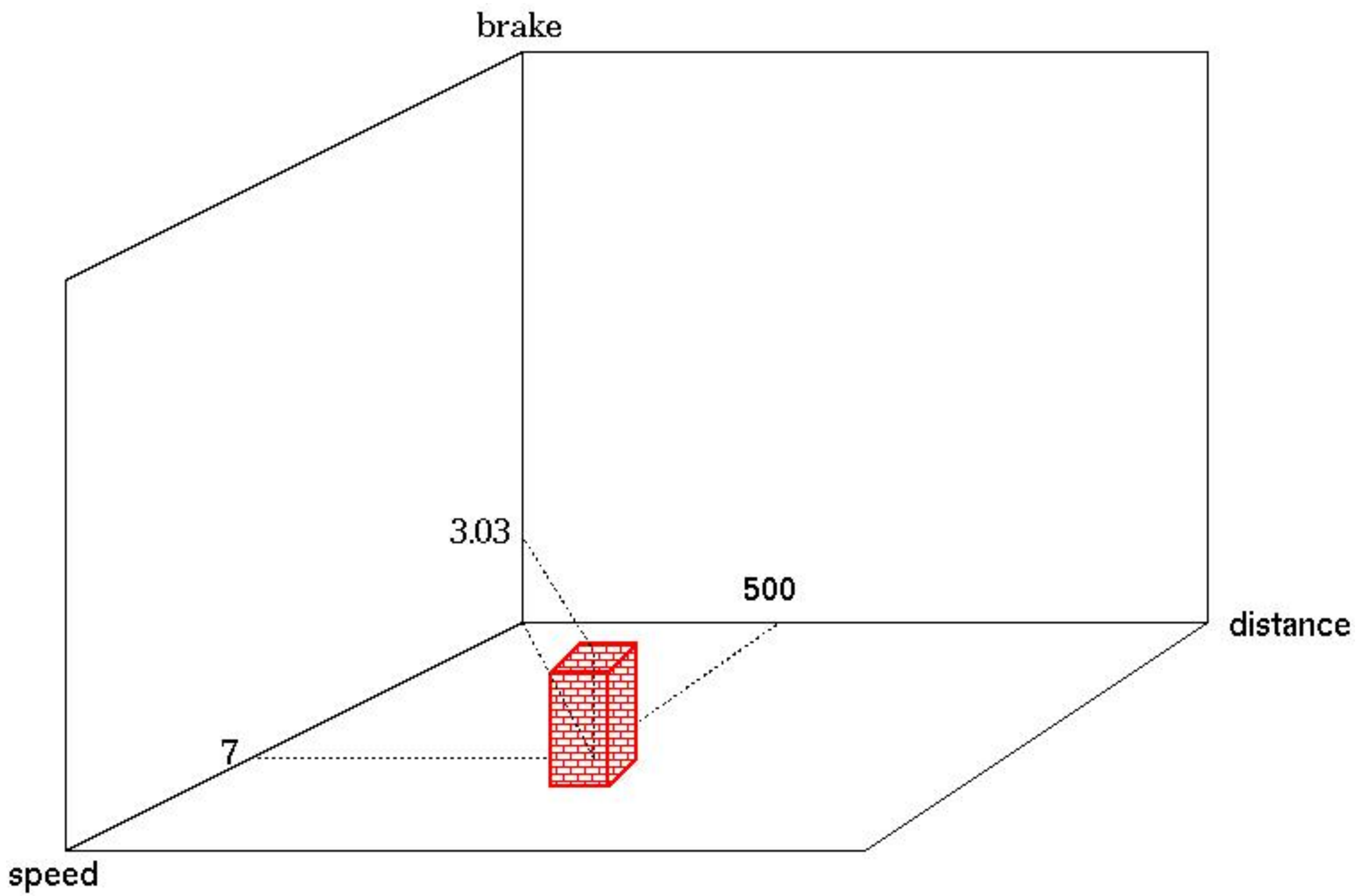
brake	0	1	2	3	4	5	6	7	8	9	10
The membership value					0.182						

	⇑	⇑	⇑	⇑		⇑	⇑	⇑	⇑	⇑	⇑
	0.000	0.000	0.175	0.350		0.135	0.000	0.000	0.000	0.000	0.000

The best appropriate value of brake?

$0.175(x-2) + 0.350(x-3) + 0.182(x-4) + 0.135(x-5) = 0 \Rightarrow 0.842x = 2.803 \Rightarrow x = 3.03$

Let’s plot one point of speed-distance-brake in the previous pagein 3D space!



What about all other combination of speed and distance?

Practice

Calculate Brake for all possible combinations of speed and distance under 2 rules in the previous page!

speed	distance	membership value of all possible brake											defuzzified brake
		o	1	2	3	4	5	6	7	8	9	10	
1	100												
	200												
	300												
	400												
	500												
	600												
	700												
	800												
	900												
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
7	100												
	200												
	300												
	400												
	500	0.000	0.000	0.175	0.350	0.182	0.135	0.000	0.000	0.000	0.000	0.000	3.03
	600												
	700												
	800												
	900												
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
9	100												
	200												
	300												
	400												
	500												
	600												
	700												
	800												
	900												

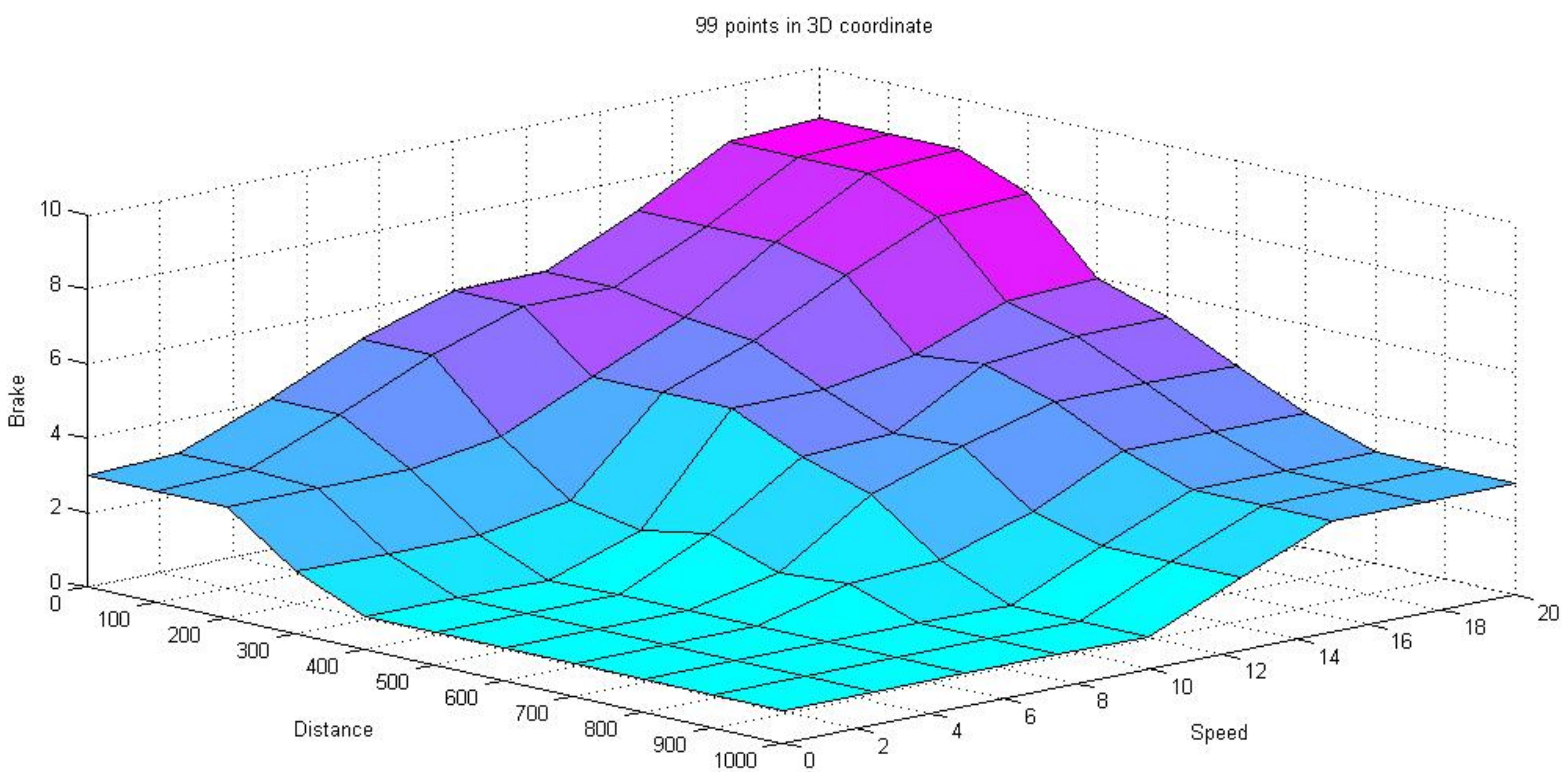
A snapshot of the table under 3 rules

By Bogutskaya Yulia (2016)

Speed	Distance	Brake	Rule 1: IF x=medium AND y=small THEN z=strong				Rule 2: IF x=medium AND y=medium THEN z=medium				Rule 3: IF x=medium AND y=large THEN z=week				Max of rules	Balance
			mSp1	mDs1	mBr1	min(mSp;mDs)*mBr	mSp2	mDs2	mBr2	min(mSp;mDs)*mBr	mSp3	mDs3	mBr3	min(mSp;mDs)*mBr		
		0	0,75	0	0	0	0,75	0,5	0	0	0,75	0,5	0	0	0	
		1	0,75	0	0	0	0,75	0,5	0	0	0,75	0,5	0	0	0	
		2	0,75	0	0	0	0,75	0,5	0	0	0,75	0,5	0,25	0,125	0,125	
		3	0,75	0	0	0	0,75	0,5	0	0	0,75	0,5	1	0,5	0,5	
		4	0,75	0	0	0	0,75	0,5	0,75	0,375	0,75	0,5	0,25	0,125	0,375	
11,00	500,00	5	0,75	0	0,3	0	0,75	0,5	0,75	0,375	0,75	0,5	0	0	0,375	3,727273
		6	0,75	0	1	0	0,75	0,5	0	0	0,75	0,5	0	0	0	
		7	0,75	0	0,3	0	0,75	0,5	0	0	0,75	0,5	0	0	0	
		8	0,75	0	0	0	0,75	0,5	0	0	0,75	0,5	0	0	0	
		9	0,75	0	0	0	0,75	0,5	0	0	0,75	0,5	0	0	0	
		10	0,75	0	0	0	0,75	0,5	0	0	0,75	0,5	0	0	0	
		0	0,75	0	0	0	0,75	0,25	0	0	0,75	0,75	0	0	0	
		1	0,75	0	0	0	0,75	0,25	0	0	0,75	0,75	0	0	0	
		2	0,75	0	0	0	0,75	0,25	0	0	0,75	0,75	0,25	0,1875	0,1875	
		3	0,75	0	0	0	0,75	0,25	0	0	0,75	0,75	1	0,75	0,75	
		4	0,75	0	0	0	0,75	0,25	0,75	0,1875	0,75	0,75	0,25	0,1875	0,1875	
11,00	550,00	5	0,75	0	0,3	0	0,75	0,25	0,75	0,1875	0,75	0,75	0	0	0,1875	3,285714
		6	0,75	0	1	0	0,75	0,25	0	0	0,75	0,75	0	0	0	
		7	0,75	0	0,3	0	0,75	0,25	0	0	0,75	0,75	0	0	0	
		8	0,75	0	0	0	0,75	0,25	0	0	0,75	0,75	0	0	0	
		9	0,75	0	0	0	0,75	0,25	0	0	0,75	0,75	0	0	0	
		10	0,75	0	0	0	0,75	0,25	0	0	0,75	0,75	0	0	0	
		0	0,75	0	0	0	0,75	0	0	0	0,75	1	0	0	0	
		1	0,75	0	0	0	0,75	0	0	0	0,75	1	0	0	0	
		2	0,75	0	0	0	0,75	0	0	0	0,75	1	0,25	0,1875	0,1875	
		3	0,75	0	0	0	0,75	0	0	0	0,75	1	1	0,75	0,75	
		4	0,75	0	0	0	0,75	0	0,75	0	0,75	1	0,25	0,1875	0,1875	
11,00	600,00	5	0,75	0	0,3	0	0,75	0	0,75	0	0,75	1	0	0	0	3
		6	0,75	0	1	0	0,75	0	0	0	0,75	1	0	0	0	
		7	0,75	0	0,3	0	0,75	0	0	0	0,75	1	0	0	0	
		8	0,75	0	0	0	0,75	0	0	0	0,75	1	0	0	0	
		9	0,75	0	0	0	0,75	0	0	0	0,75	1	0	0	0	
		10	0,75	0	0	0	0,75	0	0	0	0,75	1	0	0	0	

3D plot of previous page

By Bogutskaya Yulia (2016)



Another example under 24 rules

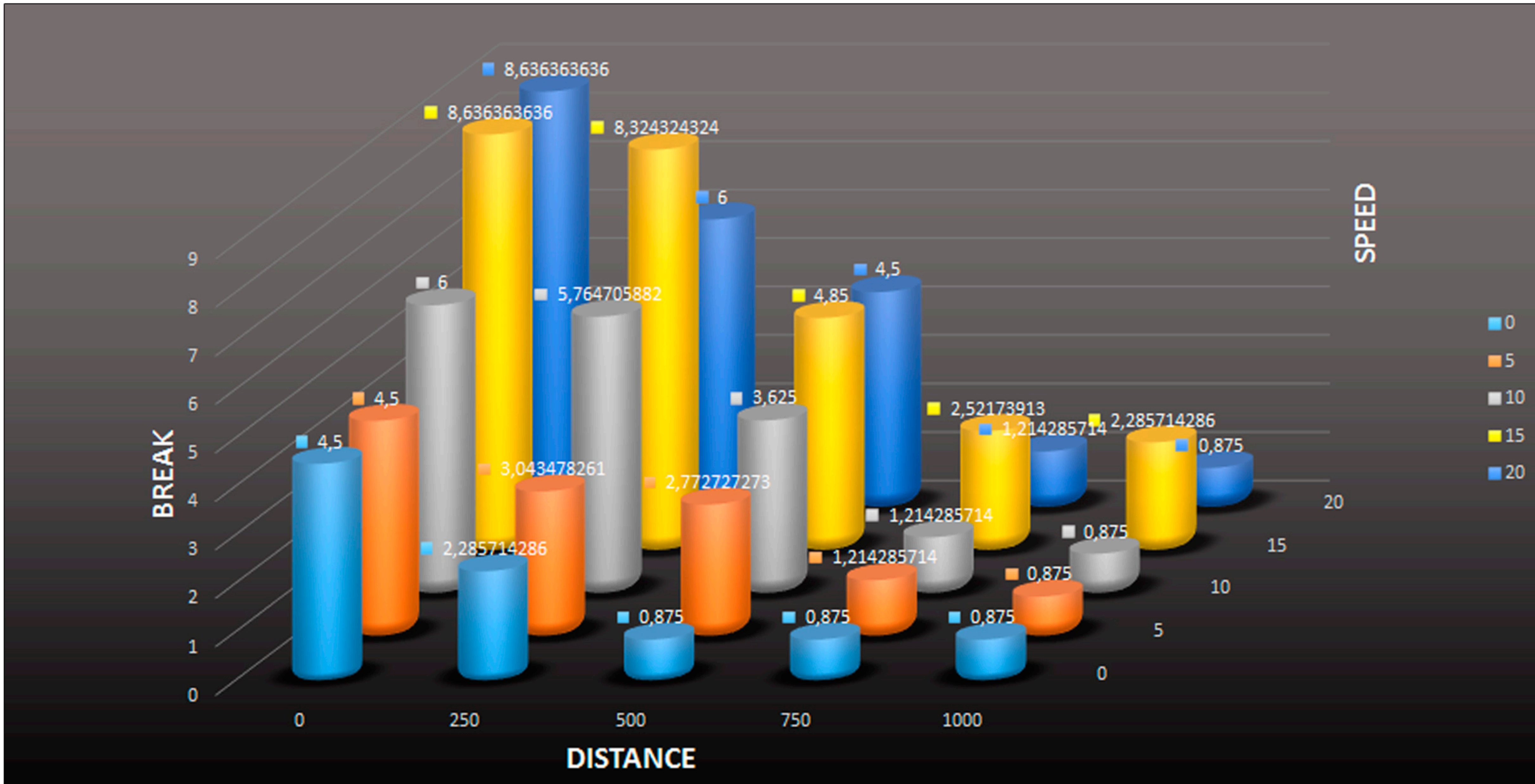
By Kurilenko Nikita (2016)

speed	0	0	0	0	0	5	5	5	5	5	10	10	10
distance	0	250	500	750	1000	0	250	500	750	1000	0	250	500
break	4,5	2,285714	0,875	0,875	0,875	4,5	3,043478	2,772727	1,214286	0,875	6	5,764706	3,625

speed	10	10	15	15	15	15	15	20	20	20	20	20
distance	750	1000	0	250	500	750	1000	0	250	500	750	1000
break	1,214286	0,875	8,636364	8,324324	4,85	2,521739	2,285714	8,636364	6	4,5	1,214286	0,875

IF speed IS	vSLOW	vSLOW	vSLOW	vSLOW	vSLOW	SLOW	SLOW	SLOW	SLOW	SLOW	MEDIUM	MEDIUM	MEDIUM
AND distance IS	vSHORT	SHORT	MEDIUM	LONG	vLONG	vSHORT	SHORT	MEDIUM	LONG	vLONG	vSHORT	SHORT	MEDIUM
THEN break IS	MEDIUM	MEDIUM	WEAK	WEAK	WEAK	MEDIUM	MEDIUM	MEDIUM	WEAK	vWEAK	STRONG	STRONG	MEDIUM

IF speed IS	MEDIUM	MEDIUM	FAST	FAST	FAST	FAST	FAST	vFAST	vFAST	vFAST	vFAST	vFAST
AND distance IS	LONG	vLONG	vSHORT	SHORT	MEDIUM	LONG	vLONG	vSHORT	SHORT	MEDIUM	LONG	vLONG
THEN break IS	WEAK	vWEAK	vSTRONG	vSTRONG	STRONG	MEDIUM	WEAK	vSTRONG	vSTRONG	STRONG	MEDIUM	WEAK



practice

How about under your own 10 rules ?

X: speed Y: distance Z_i : defuzzified brake Z: brake

X	Y	Rule 1												Rule 2													Rule 10												Max Z_i = Z
		$\mu_1(Z)$											Z_1	$\mu_2(Z)$											Z_2		$\mu_{10}(Z)$											Z_{10}	
		0	1	2	3	4	5	6	7	8	9	10		0	1	2	3	4	5	6	7	8	9	10			0	1	2	3	4	5	6	7	8	9	10		
	100																																						
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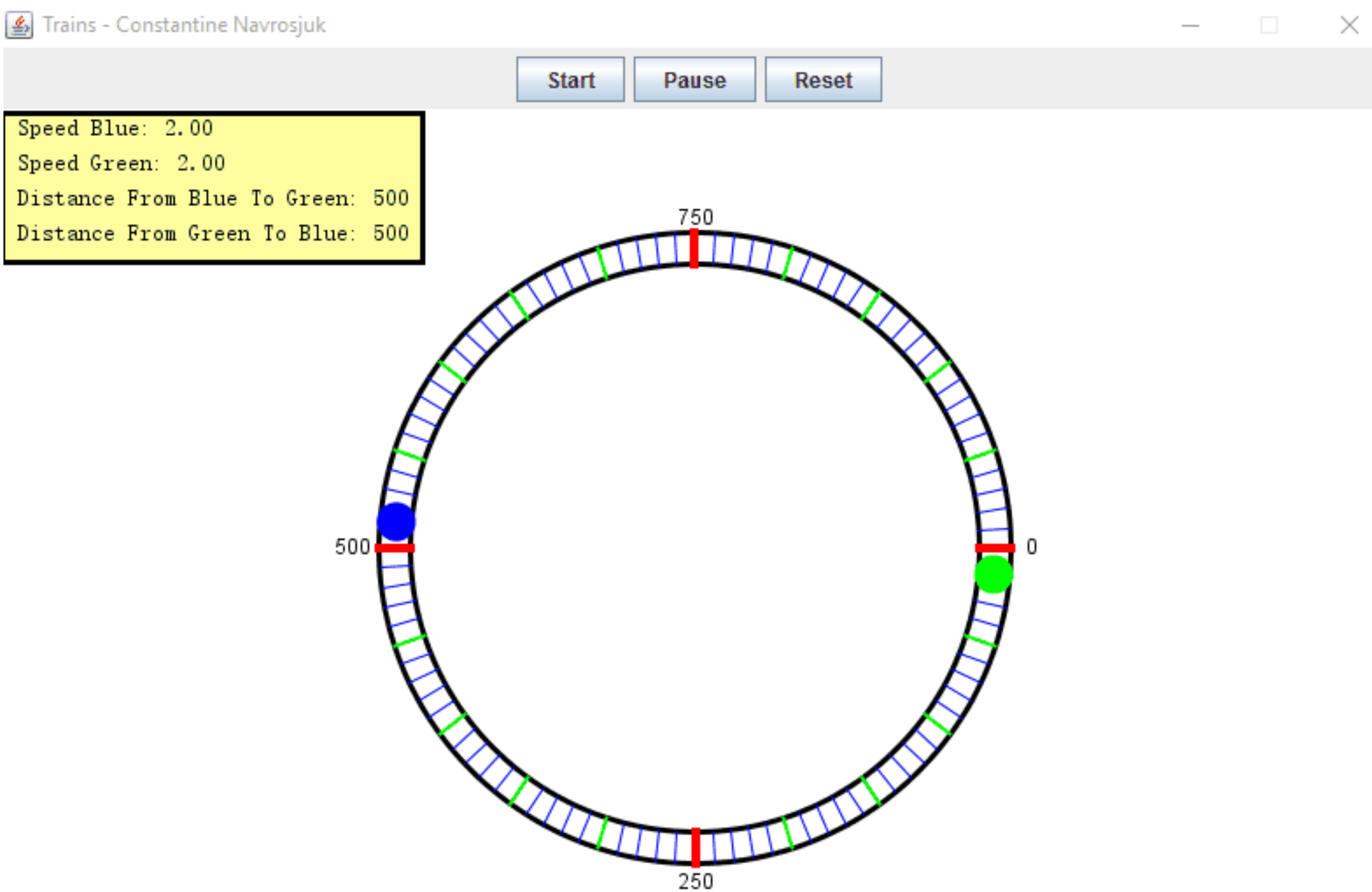
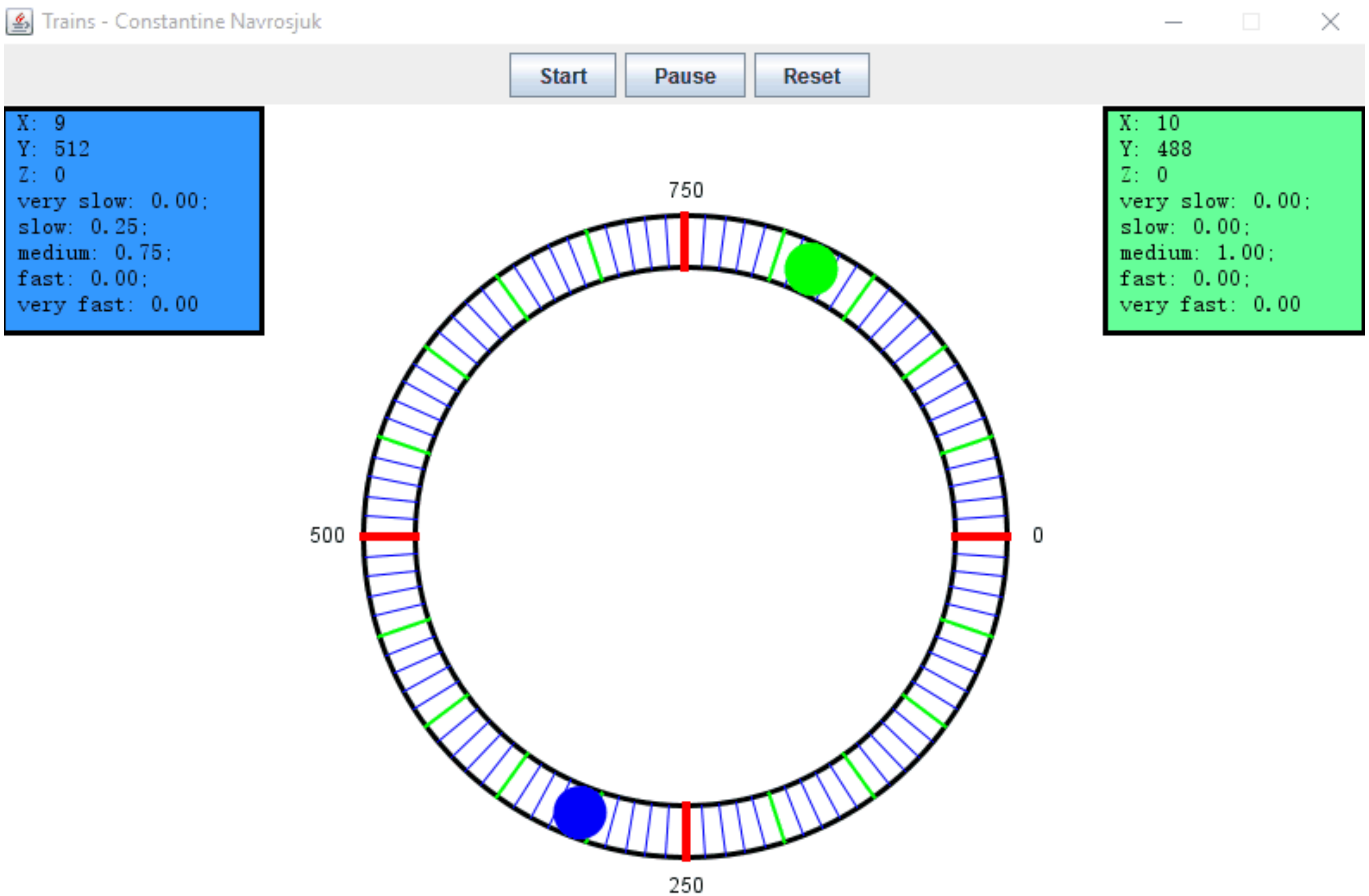
Two metro cars in one loop with constant speed - animation

By Navrosjuk Kostia (2016)



Two metros in one loop when speed changes at random

By Navrosjuk Kostia (2016)



Then avoid crash of two metro cars
by using appropriate value of your own
10 rules in each step!

II. Classify Iris Flowers!

Iris Flower Database to design

Setosa				Versicolor				Virginica			
x_1	x_2	x_3	x_4	x_1	x_2	x_3	x_4	x_1	x_2	x_3	x_4
0.56	0.66	0.20	0.08	0.84	0.66	0.67	0.52	0.85	0.57	0.84	0.72
0.62	0.70	0.22	0.04	0.66	0.61	0.57	0.56	0.91	0.82	0.88	1.00
0.68	0.84	0.22	0.08	0.63	0.45	0.51	0.40	0.82	0.73	0.74	0.80
0.61	0.77	0.23	0.08	0.75	0.68	0.61	0.60	0.81	0.61	0.77	0.76
0.61	0.68	0.20	0.04	0.76	0.50	0.58	0.40	0.86	0.68	0.80	0.84
0.54	0.68	0.16	0.04	0.77	0.66	0.68	0.56	0.72	0.57	0.72	0.80
0.73	0.91	0.17	0.08	0.71	0.66	0.52	0.52	0.73	0.64	0.74	0.96
0.72	1.00	0.22	0.16	0.85	0.70	0.64	0.56	0.81	0.73	0.77	0.92
0.68	0.89	0.19	0.16	0.71	0.68	0.65	0.60	0.82	0.68	0.80	0.72
0.65	0.80	0.20	0.12	0.73	0.61	0.59	0.40	0.97	0.86	0.97	0.88
0.72	0.86	0.25	0.12	0.78	0.50	0.65	0.60	0.97	0.59	1.00	0.92
0.65	0.86	0.22	0.12	0.71	0.57	0.57	0.44	0.76	0.50	0.72	0.60
0.68	0.77	0.25	0.08	0.75	0.73	0.70	0.72	0.87	0.73	0.83	0.92
0.65	0.84	0.22	0.16	0.77	0.64	0.58	0.52	0.71	0.64	0.71	0.80
0.58	0.82	0.14	0.08	0.80	0.57	0.71	0.60	0.97	0.64	0.97	0.80
0.65	0.75	0.25	0.20	0.77	0.64	0.68	0.48	0.80	0.61	0.71	0.72
0.61	0.77	0.28	0.08	0.81	0.66	0.62	0.52	0.85	0.75	0.83	0.84

The original $x1$ values of 3 families of iris flower
and
determination the range of Large, Medium and Small.

Original data of x_1

Setosa	Versicolor	Virginica
0.56	0.84	0.85
0.62	0.66	0.91
0.68	0.63	0.82
0.61	0.75	0.81
0.61	0.76	0.86
0.54	0.77	0.72
0.73	0.71	0.73
0.72	0.85	0.81
0.68	0.71	0.82
0.65	0.73	0.97
0.72	0.78	0.97
0.65	0.71	0.76
0.68	0.75	0.87
0.65	0.77	0.71
0.58	0.80	0.97
0.65	0.77	0.80
0.61	0.81	0.85

=>

Sort

all data in 3 columns

with descending order

and then

devided into 3 category

0.97 ··· 0.80 0.78 ··· 0.71 0.68 ··· 0.64

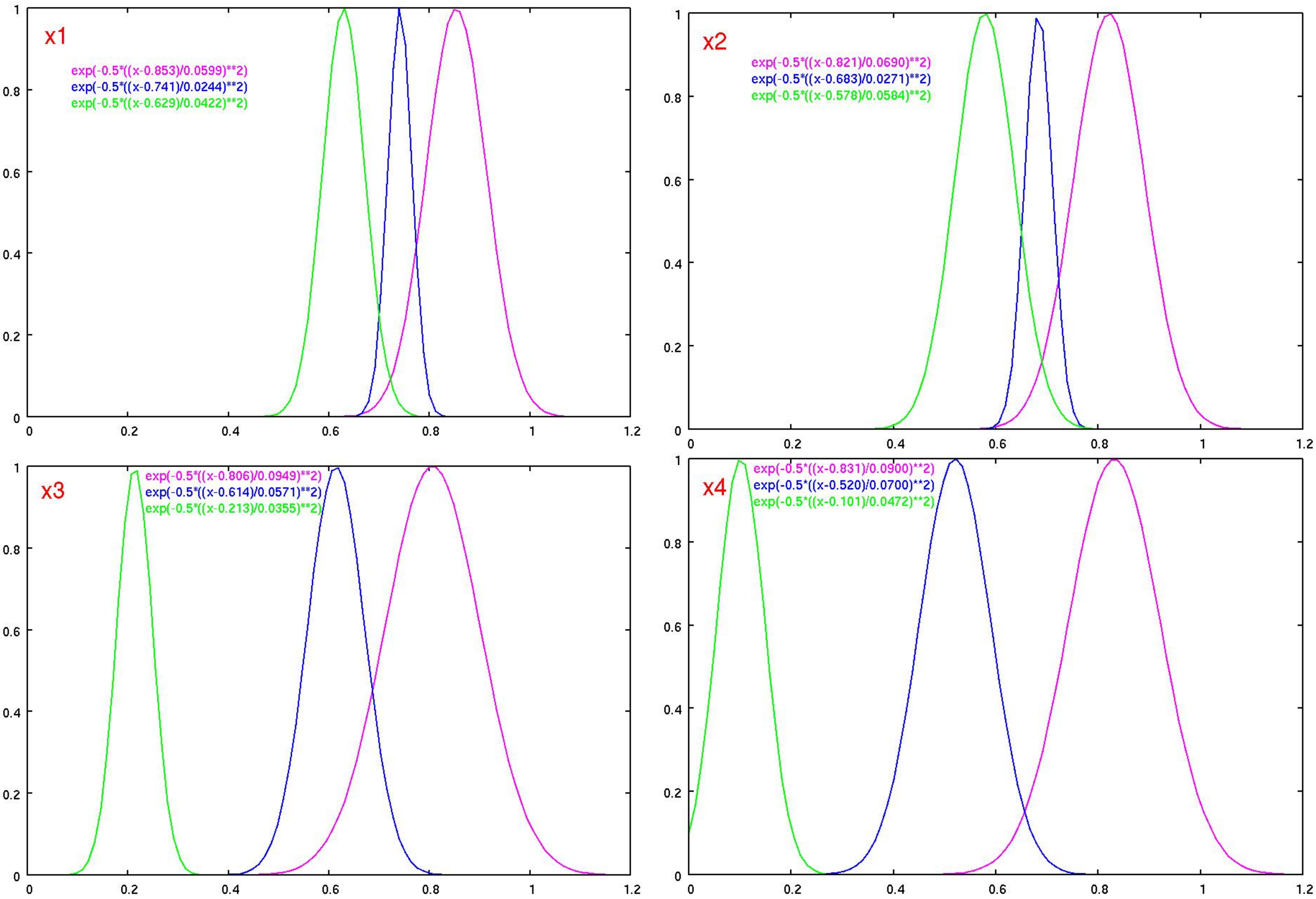
Large Medium short

	Large	Medium	short
	0.97	0.78	0.68
	0.97	0.77	0.68
	0.97	0.77	0.68
	0.91	0.77	0.66
	0.87	0.76	0.65
	0.86	0.76	0.65
	0.85	0.75	0.65
	0.85	0.75	0.66
	0.85	0.73	0.63
	0.84	0.73	0.62
	0.82	0.73	0.61
	0.82	0.72	0.61
	0.81	0.72	0.61
	0.81	0.72	0.58
	0.81	0.71	0.56
	0.80	0.71	0.64
	0.80	0.71	
		0.71	
avg	0.853	0.741	0.629
std	0.0599	0.0244	0.0422

In this way we get:

		Large	Medium	short
x1		0.97 ··· 0.80	0.78 ··· 0.71	0.68 ··· 0.64
	avg	0.853	0.741	0.629
	std	0.0599	0.0244	0.0422
x2		1.00 ··· 0.75	0.73 ··· 0.66	0.64 ··· 0.45
	avg	0.821	0.683	0.578
	std	0.0690	0.0271	0.0584
x3		1.00 ··· 0.71	0.70 ··· 0.51	0.28 ··· 0.14
	avg	0.806	0.613	0.213
	std	0.0949	0.0571	0.0355
x4		1.00 ··· 0.72	0.60 ··· 0.40	0.20 ··· 0.04
	avg	0.831	0.520	0.101
	std	0.0900	0.0700	0.0472

Membership function of Small, Medium and Large for x1, x2, x3, and x4



Now let’s translate numerical values into human language

Setosa				Versicolor				Virginica			
x1	x2	x3	x4	x1	x2	x3	x4	x1	x2	x3	x4
small	medium	small	small	large	medium	medium	medium	large	small	large	large
small	medium	small	small	small	medium	medium	medium	large	large	large	large
small	large	small	small	small	large	medium	medium	large	medium	large	large
small	large	small	small	medium	large	medium	medium	large	small	large	large
small	medium	small	small	medium	medium	medium	medium	large	medium	large	large
small	medium	small	small	medium	medium	medium	medium	large	small	large	large
medium	large	small	small	large	large	medium	medium	medium	small	large	large
medium	large	small	small	medium	large	medium	medium	medium	medium	large	large
small	large	small	small	small	large	medium	medium	large	medium	large	large
small	large	small	small	small	large	medium	medium	large	large	large	large
medium	large	small	small	medium	large	medium	medium	large	small	large	large
small	large	small	small	medium	large	medium	medium	large	medium	large	large
small	large	small	small	medium	large	medium	medium	medium	medium	large	medium
small	large	small	small	medium	large	medium	large	large	medium	large	large
small	large	small	small	medium	large	medium	medium	medium	small	large	large
small	large	small	small	large	large	large	medium	large	small	large	large
small	large	small	small	medium	large	medium	medium	large	small	large	large
small	large	small	small	large	large	medium	medium	large	large	large	large

Rules to classify iris flowers

E.g.

R_1: IF x1 = $\boxed{\text{small}}$ AND x2 = $\boxed{\text{large}}$ AND x3 = $\boxed{\text{small}}$ AND x4 = $\boxed{\text{small}}$ THEN y = 1

OR

R_2: IF x1 = $\boxed{\text{medium}}$ AND x2 = $\boxed{\text{large}}$ AND x3 = $\boxed{\text{medium}}$ AND x4 = $\boxed{\text{medium}}$ THEN y = 2

OR

R_3: IF x1 = $\boxed{\text{large}}$ AND x2 = $\boxed{\text{small}}$ AND x3 = $\boxed{\text{large}}$ AND x4 = $\boxed{\text{large}}$ THEN y = 3

OR

R_4: IF x1 = $\boxed{\text{large}}$ AND x2 = $\boxed{\text{medium}}$ AND x3 = $\boxed{\text{large}}$ AND x4 = $\boxed{\text{large}}$ THEN y = 3

which family the next irises belongs to?

x1 = 0.80, x2 = 0.75, x3 = 0.87 and x4 = 1.00

	x1	x2	x3	x4	y
R1:	small	large	small	small	1
R2:	medium	large	medium	medium	2
R3:	large	small	large	large	3
R4:	large	medium	large	large	3

$$y = \{ \Sigma_{k=1}^{H} \; (M_k(x) \cdot g_k \} / \{ \Sigma_{k=1}^{H} \; (M_k(x) \}$$ (H = 4 and j = 1,2,3,4)

where

$$M_k(x) = \prod_{i=1}^{M} \mu_{ik}(x_i)$$ (M = 4)

k = index of rule, H = number of rule, M = number of attribute

	X1	X2	X3	X5
Large:	exp(-0.5((x-0.853)/0.0599)**2)	exp(-0.5((x-0.821)/0.0690)**2)	exp(-0.5((x-0.806)/0.0949)**2)	exp(-0.5((x-0.831)/0.0900)**2)
Medium:	exp(-0.5((x-0.741)/0.0244)**2)	exp(-0.5((x-0.683)/0.0271)**2)	exp(-0.5((x-0.614)/0.0571)**2)	exp(-0.5((x-0.520)/0.0700)**2)
Small:	exp(-0.5((x-0.629)/0.0422)**2)	exp(-0.5((x-0.578)/0.0584)**2)	exp(-0.5((x-0.213)/0.0355)**2)	exp(-0.5((x-0.101)/0.0472)**2)

μ_{11} = exp(-0.5 ((0.80 -0.629)/0.0422)^2)=0.000 : μ_{12} = exp(-0.5 ((0.75 -0.821)/0.0690)^2)=0.589: μ_{13} = exp(-0.5 ((0.87 -0.213)/0.0355)^2)=0.000: μ_{14} = exp(-0.5 ((1.00 -0.101)/0.0472)^2)=0.000
μ_{21} = exp(-0.5 ((0.80 -0.741)/0.0244)^2)=0.054 : μ_{22} = exp(-0.5 ((0.75 -0.821)/0.0690)^2)=0.589: μ_{23} = exp(-0.5 ((0.87 -0.614)/0.0571)^2)=0.000: μ_{24} = exp(-0.5 ((1.00 -0.520)/0.0700)^2)=0.000
μ_{31} = exp(-0.5 ((0.80 -0.853)/0.0599)^2)=0.068 : μ_{32} = exp(-0.5 ((0.75 -0.578)/0.0584)^2)=0.013: μ_{33} = exp(-0.5 ((0.87 -0.806)/0.0949)^2)=0.797: μ_{34} = exp(-0.5 ((1.00 -0.831)/0.0900)^2)=0.172
μ_{41} = exp(-0.5 ((0.80 -0.853)/0.0599)^2)=0.068 : μ_{42} = exp(-0.5 ((0.75 -0.683)/0.0271)^2)=0.047: μ_{43} = exp(-0.5 ((0.87 -0.806)/0.0949)^2)=0.797: μ_{44} = exp(-0.5 ((1.00 -0.831)/0.0900)^2)=0.172

M1 = 0.000x0.589x0.000x0.000 = 0.00000000
M2 = 0.054x0.589x0.000x0.000 = 0.00000000
M3 = 0.068x0.013x0.797x0.172 = 0.00012118
M4 = 0.068x0.047x0.797x0.171 = 0.00043812

y = (0.0000000x1+0.000000x2+0.00012118x3+0.00043812x3) / (0.0000000+0.00000000+0.00012118+0.00043812
= 0.00167790 / 0.0005593
= 3.0

Banana dataset

(extracted from UCI (University of California, Irvine) Machine Learning Repository)

A		B	
x_1	x_2	x_1	x_2
-1.520	-1.150	1.140	-0.114
-0.916	0.397	-1.050	0.720
-1.090	0.437	1.830	0.452
-0.584	0.094	1.790	-0.459
-1.250	-0.286	-0.122	-0.808
1.700	1.210	-0.768	-1.040
-0.482	-0.485	0.724	0.989
0.081	1.930	0.444	1.990
-0.541	-0.332	-1.010	-1.360
-1.690	-1.150	1.280	0.691
1.260	1.210	0.925	0.895
-0.863	0.496	-0.687	-1.290
1.160	0.458	1.710	-0.044
-0.595	-0.651	1.120	0.626
-0.770	0.364	1.300	0.196
-0.871	-0.825	1.130	1.480
0.996	-1.700	0.763	0.921
1.740	0.964	-1.410	1.110
1.180	-0.335	-0.750	-0.881
2.520	1.430	1.116	0.978
0.271	-0.591	1.130	0.405
-1.590	--0.68	-0.522	-1.340
0.408	0.067	-1.310	1.250
0.009	-0.434	0.041	-1.130
-2.140	-1.430	0.048	0.866
0.007	0.012	-2.110	0.193
-0.352	-0.490	0.522	1.460
1.330	1.510	0.028	1.620
1.090	-1.370	0.536	0.921
-1.670	-1.260	-0.123	-1.070
-0.508	-9.715	0.526	1.480

- Sort values in both of A and B respectively
- Divide these sorted values into 3 groups - large, medium & small
- Calculate avg & std of these 6 groups
- Draw Gaussian membership functions for each of these 6 groups
- Translate all numerical values into natural language: large, medium & small
- Create rules to classify data
- Then guess that the data [x1 = -1.620 & x2 =0.468 is class A or B

Mammo graphic dataset

(extracted from UCI (University of California, Irvine) Machine Learning Repository)

Normal

X_1	X_2	X_3	X_4	X_5
5	57	3	5	3
5	58	4	5	3
5	57	1	5	3
5	76	1	4	3
3	42	2	1	3
4	59	2	1	3
4	54	1	1	3
5	56	4	3	1
5	42	4	4	3
4	59	2	4	3
5	75	6	5	3
6	71	4	4	3
5	62	3	5	2
5	80	3	5	3
5	74	1	1	2

Not normal

X_1	X_2	X_3	X_4	X_5
4	28	1	1	3
4	36	3	1	2
4	60	2	1	2
4	54	1	1	3
3	52	3	4	3
5	86	4	4	3
5	66	4	4	4
5	60	3	1	3
3	45	2	1	3
3	43	2	1	3
2	49	2	1	3
4	47	3	1	3
4	24	2	1	3
6	41	2	1	3
4	19	1	1	3

X_1: BI-RADS, X_2: Age, X_3: Shape, X_4 : Margin, X_5 : Density

- Sort values in both Normal and Not normal
- Divide these sorted values into 2 groups - Large & Small
- Calculate avg & std of these 10 groups
- Draw Gaussian membership functions of each of these 10 groups
- Translate all numerical values into Large or Small
- Create rules to classify data
- Then guess that data (4, 62, 3, 3, 3) is Normal or Not normal