Error Correcting Capability of a Morphological Associative Memory

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- Traditional Associative Memory

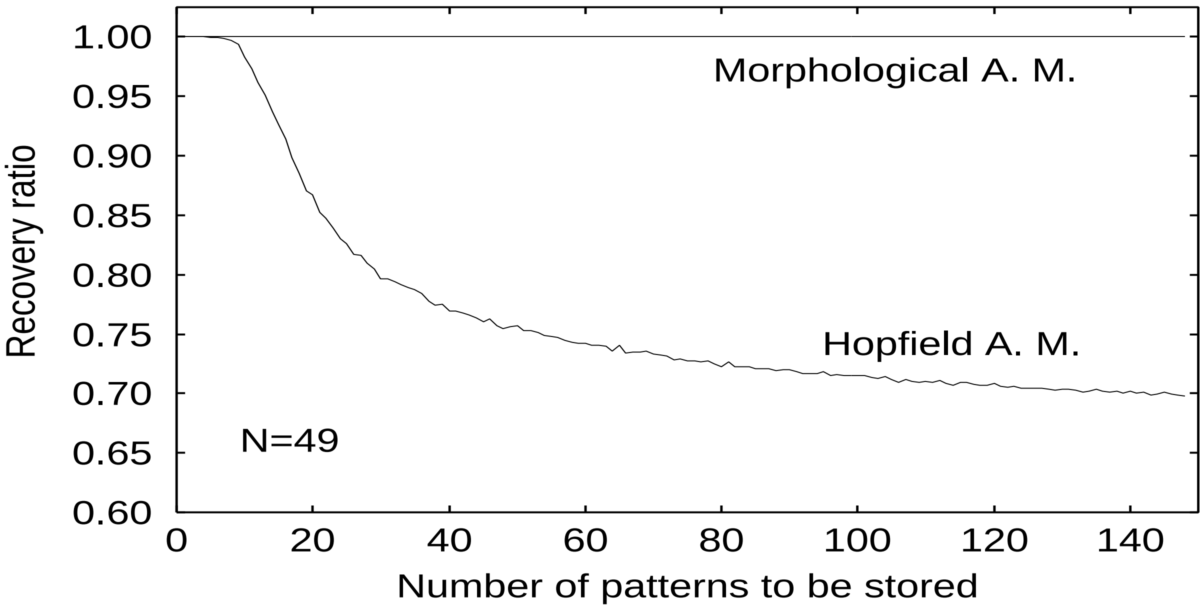
  $p < 0.138N \cdots$ Hebb's rule (1982)

  $p < 2N \cdots$ Gardner (1988)

- Morphological Associative Memory

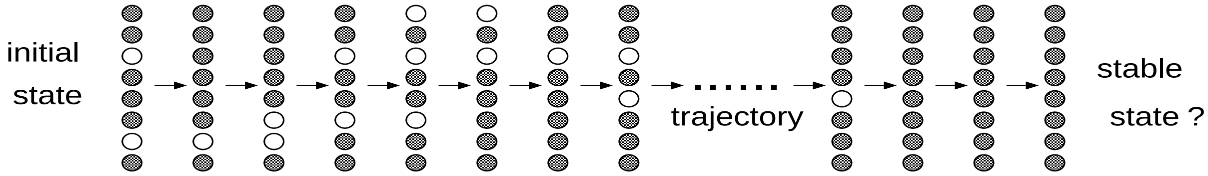
  $p < 2^N$

- **Infinitely large storage capacity!**



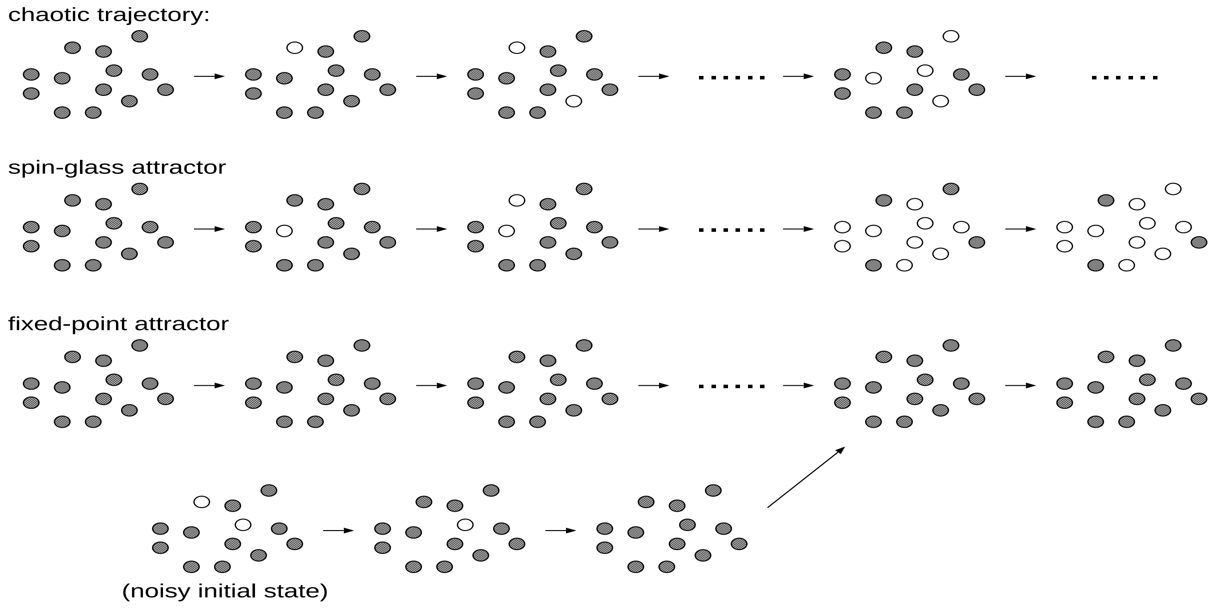
- **But how about the size of basin-of-attraction?**

associative memory = dynamical system



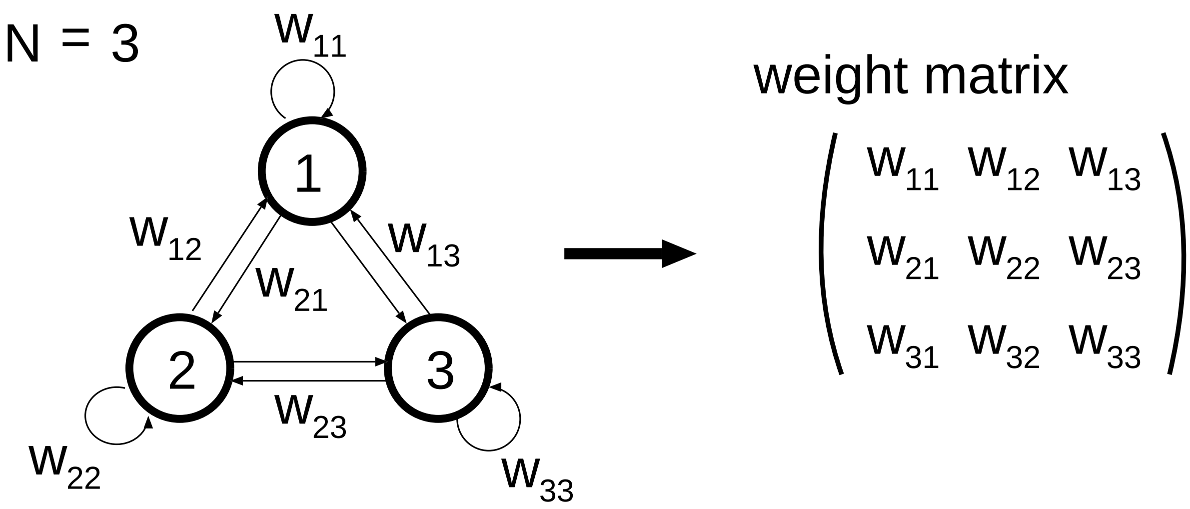
initial state $\xrightarrow{\text{converge}}$ equilibrium state

u+du $\qquad$ u


chaotic trajectory:
spin-glass attractor
fixed-point attractor
(noisy initial state)

□The Associative Memory can be realized with:

- fully-connected neural network model;
- immune system model;
- etc.



N = 3
w11
w12
w13
w21
w23
w22
w33
1
2
3
weight matrix
$$\begin{pmatrix} w_{11} & w_{12} & w_{13} \\ w_{21} & w_{22} & w_{23} \\ w_{31} & w_{32} & w_{33} \end{pmatrix}$$

□ To store patterns

$$(\xi_1^\mu, \xi_2^\mu, \cdots, \xi_N^\mu), \quad \mu = 1, 2, \cdots, p$$

□ Hopfield Model

- Hebbian weights:

$$w_{ij} = \frac{1}{N} \sum_{\nu=1}^{p} \xi_i^\nu \xi_j^\nu \ \ (i \neq j), \ \ w_{ii} = 0$$

- Rule for update:

$$s_i(t+1) = sgn\left(\sum_{j \neq i}^{N} w_{ij} s_j(t) \right)$$

□ Morphological Model

- Weights:

$$w_{ij} = \min_{\nu=1}^{p}(\xi_i^{\nu} - \xi_j^{\nu})$$

- Rule for update

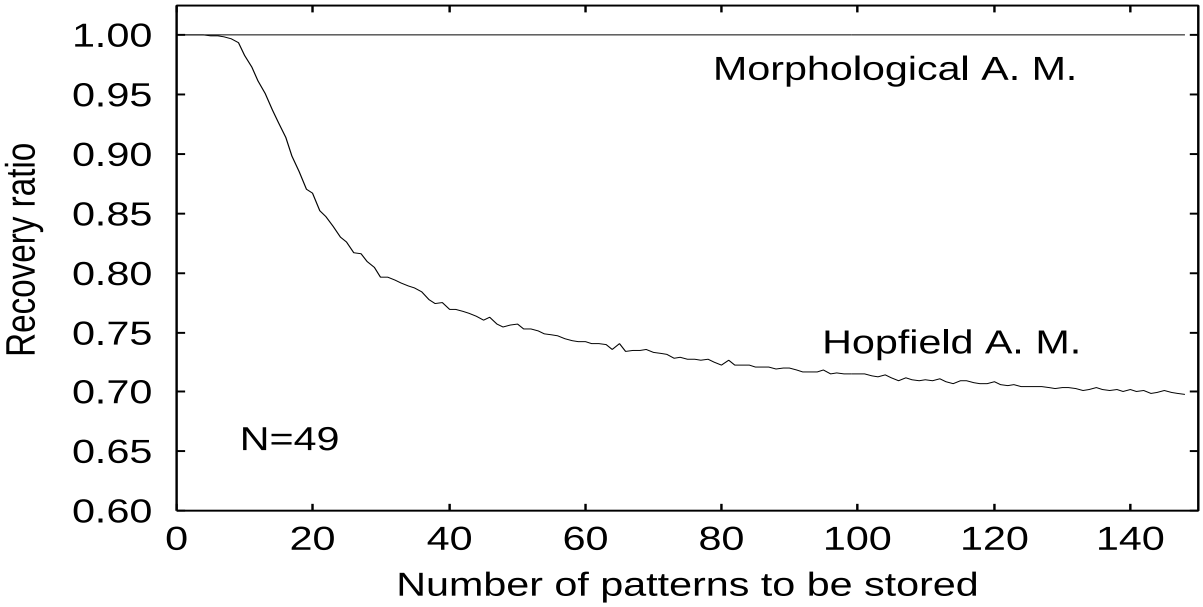
$$s_i(t+1) = sgn\left(\max_{j=1}^{N}(w_{ij} + s_j(t))\right)$$

□ An assertion in Morphological Model:

- “It has an unlimited storage capacity.”

□ **Recovery ratio:**

- Average number of bits of converged patterns that are unchanged from the input.



1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.60
Recovery ratio
Morphological A. M.
Hopfield A. M.
N=49
0
20
40
60
80
100
120
140
Number of patterns to be stored

□ Error Correcting Capability:

- A noisy input of a stored pattern converges to the perfect version of the pattern, if the number of noisy bits is within the basin of attraction.
  - Hopfield Model:
    - ⋆ in several steps; or
    - ⋆ sometimes via a long long tour.
  - Morphological Model:
    - ⋆ in one step for any input pattern.

□ Errors in the Morphological Model:

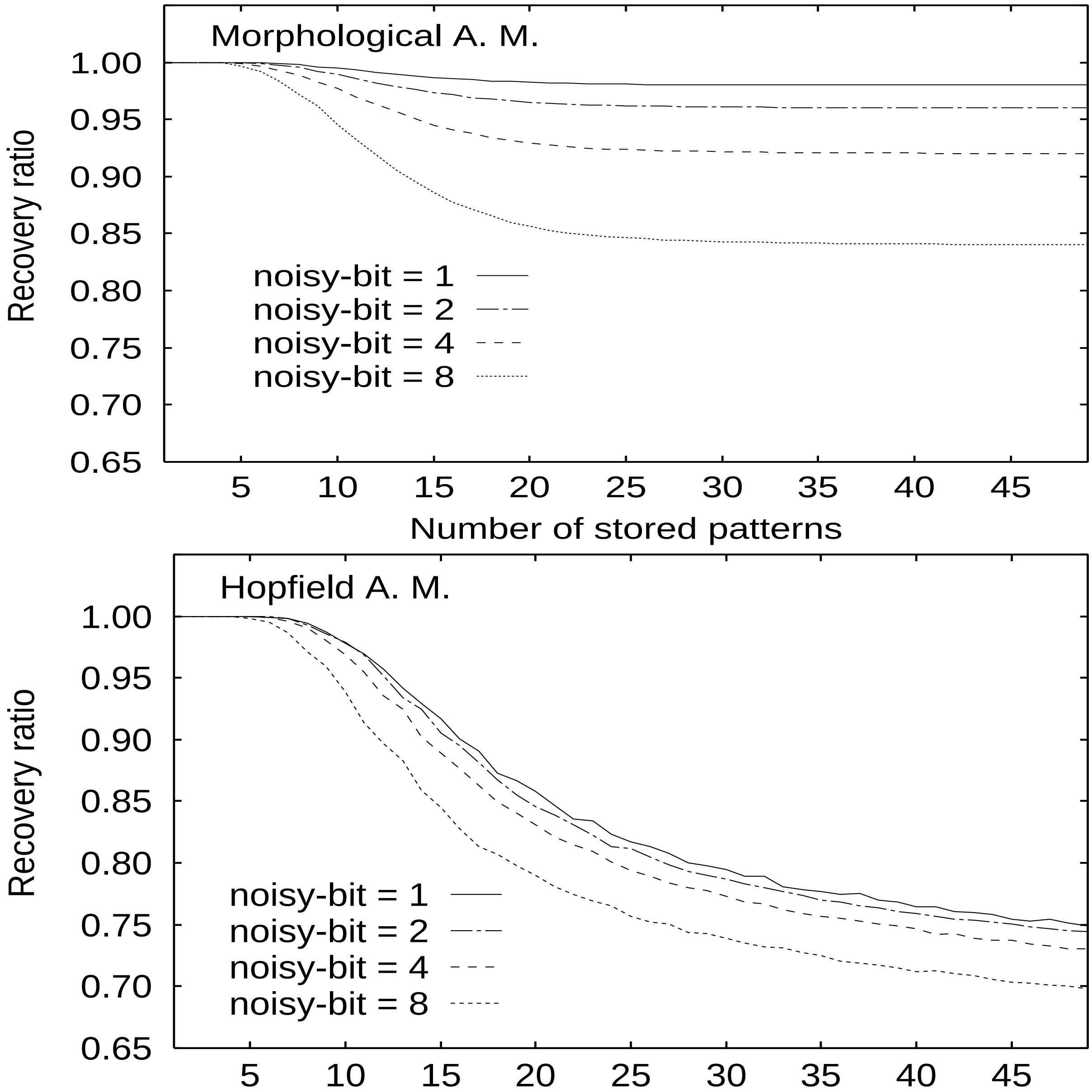
- erosive error

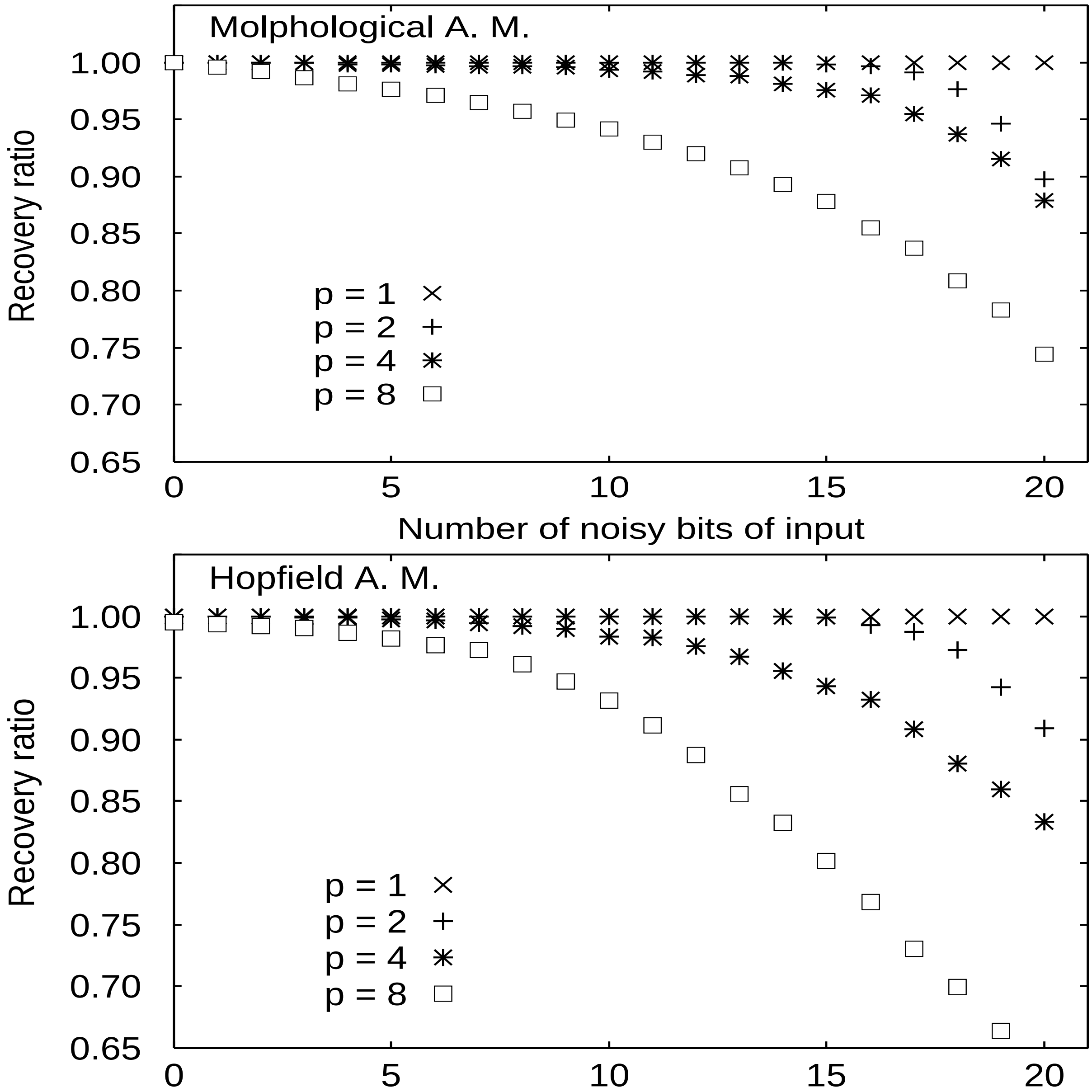
    that changes some of $+1$ of the input pattern into -1,

- dilative error

    that changes some of -1 into $+1$.

are processed by two different modules, though we focus only on erosive errors here.


Morphological A. M.
Recovery ratio
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
5
10
15
20
25
30
35
40
45
Number of stored patterns
noisy-bit = 1
noisy-bit = 2
noisy-bit = 4
noisy-bit = 8
Hopfield A. M.
Recovery ratio
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
5
10
15
20
25
30
35
40
45
noisy-bit = 1
noisy-bit = 2
noisy-bit = 4
noisy-bit = 8


Molphological A. M.
Recovery ratio
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0
5
10
15
20
Number of noisy bits of input
p = 1
p = 2
p = 4
p = 8
Hopfield A. M.
Recovery ratio
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0
5
10
15
20
p = 1
p = 2
p = 4
p = 8

□ Conclusions:

- Storage capacity of morphological associative memory is very large.
- But the basin size is suffered from the large storage more or less as the Hopfield Model.
- Can we apply it to real-world problems?