ERROR CORRECTING CAPABILITY OF A MORPHOLOGICAL ASSOCIATIVE MEMORY

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Abstract-*Morphological associative memory is a neural network model of associative memory which was proposed with an assertion that it could store as much as 2^N patterns, while the traditional Hopfield scheme, for instance, can store $2N$ patterns at most, where N is the number of neurons. However, what will happen to its error correcting capability when we are to store as many patterns as this upper bound of the capacity in a morphological associative memory? The error correcting capability of the morphological associative memory model has not been studied extensively. In this paper, we explore this issue by comparing it with the fairly well studied Hopfield model.*

1 INTRODUCTION

Associative memory is a dynamical system which has a number of stable states with each state having a domain of attraction [?]. If the system starts at any state in a domain, it will converge to the stable state.

The associative memory has been realized mainly with a fully connected neural network model. Provided the synaptic strengths among neurons are determined appropriately, network stores a number of patterns as attractors. An initial network state which is possibly a noisy and/or partial input of one of these stored patterns is updated repeatedly and preferably converges to the corresponding pattern. If an initial input in the vicinity of one of the memory converges to the memory, the memory is said to be recalled with the error being corrected.

In 1982, Hopfield [?] proposed a model in which he used the so-called Hebb's [?] rule to determine the synaptic strengths, as well as an asynchronous rule to update network state. Since then, a lot of other schemes of determining weights and updating state have been proposed. In these schemes, storage capacity is traded off against error correcting capability, and Gardner [?] analytically proved that the error correcting capability shrinks to zero when the number of stored patterns approaches the number twice as much as the number of neurons.

Morphological associative memory is proposed by Ritter, Susner, and Diaz-de-Leon [?, ?], to overcome the limit of this storage capacity, as they described "*it has an unlimited storage capacity.*" The idea of morphological neural networks *grew out of the theory of image algebra* [?], and *the operations are based on the mathematical morphology* [?], as cited in [?, ?]. Since the error correcting capability of the morphological associative memory was not described in [?, ?], we explore it in this paper by comparing it with the Hopfield model.

2 HOPFIELD'S VERSUS MORPHOLOGICAL

The neural network model of associative memory, in general, consists of N neurons and N^2 synapses. Each neuron state is either active (+1) or quiescent (−1). When an arbitrary N-bit *bipolar pattern*, a sequence of +1 and −1,

$$\xi^{\nu} = (\xi_1^{\nu}, \cdots, \xi_N^{\nu}), \qquad \nu = 1, \cdots, p, \tag{1}$$

is given to the network as an initial state, the dynamical behavior of neurons' states afterwards are characterized by the strengths of the N^2 synapses. The synaptic strengths are called *weights*, and the weight from neuron j to neuron i is denoted as w_{ij} in this paper.

2.1 Hopfield Model

In order to store p patterns described in equation (**??**), Hopfield used the so-called Hebb's rule to determine weight values, as follows,

$$w_{ij} = \frac{1}{N} \sum_{\nu=1}^{p} \xi_i^{\nu} \xi_j^{\nu} \quad (i \neq j), \quad w_{ii} = 0. \tag{2}$$

Further, an instantaneous state of a neuron is updated asynchronously (one neuron at a time) as:

$$s_i\,(t+1) = sgn\left(\sum_{j \neq i}^{N} w_{ij} s_j(t)\right), \tag{3}$$

where $s_i(t)$ is a state of the i-th neuron at time t. Then the p patterns are stored as fixed point attractors, if p is sufficiently small.

2.2 Morphological Model

As mentioned earlier, associative memory should be robust for input noises. That is, small errors given to an input of the stored patterns should be corrected when the network converged to the stable state. In the morphological associative memory, noises are devided into two categories: "*erosive errors*" and "*dilative errors*". Erosive error changes some of +1 of the input pattern into −1, and dilative error changes some of −1 into +1. Each of these two noises is processed by different module. For the sake of simplicity, however, our focus will be only on erosive errors in this paper. To store a set of patterns (**??**), the weight values are determined by the following way.

$$w_{ij} = \min_{\nu=1}^{p}(\xi_i^{\nu} - \xi_j^{\nu}), \tag{4}$$

and update of neurons' state is as follows.

$$s_i\,(t+1) = sgn\left(\max_{j=1}^{N}(w_{ij} + s_j(t))\right). \tag{5}$$

As described in [**?**, **?**], it is interesting to compare (**??**) and (**??**) with (**??**) and (**??**), respectively, and to notice its duality.

In their paper, Ritter et al. [**?**], proved that *"the memories converge in one step for any input pattern,"* which is very different from the Hopfield model where we sometimes have to wait for a long time until an initial state converges to a stable state.

3 RESULTS AND DISCUSSION

Comparisons are made using networks with 49 neurons. By "noise" in this section we mean dilative noise, i.e., noises are given to some of "+1" bits turning them into "−1" bits.

3.1 Storage as fixed points

Associative memory stores a number of patterns as fixed point attractors. That is, any input of the stored patterns remains unchanged. However, when the number of patterns to be stored increases (over-loaded), this capability will be corrupted, in general. Some of the over-loaded patterns result in a transition of the initial states, that is, the initial state converges to some other point close to the memorized pattern, or diverges. So, first of all, we compare the storage capacity as fixed point attractors of the two models, by observing an average correct bit ratio of recalled patterns when one of stored patterns is given without any noise.

The results are shown in Figure 1. As Ritter et al. [?] described the morphological associative memory stored patterns perfectly as fixed points. We observed it even if the number of storage exceeds 140, while the Hopfield associative memory fails it for more than 16 patterns.

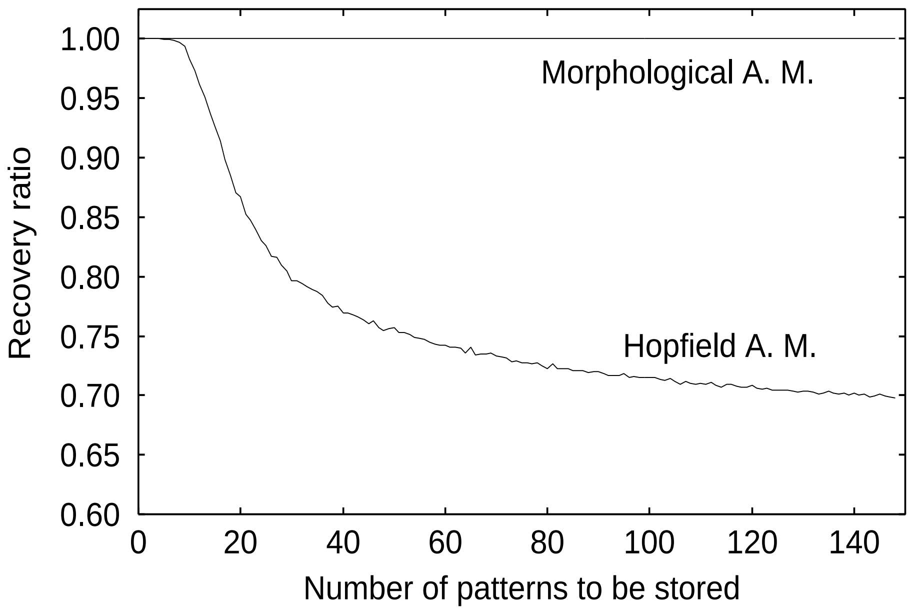

Figure 1: Average correct bit ratio of recalled patterns when stored patterns are given without noise.

3.2 Basin size of attraction

Then comparisons are made with noisy inputs. Figure 2 shows the recovery ratio as a function of number of noisy bit of input patterns, that is, size of basin of attraction. We observed it when the number of storage is 1, 2, 4, and 8.

When the number of patterns to be stored is only one, the task is easy, and the both models recalled the patterns perfectly even for the 20 eluded bits. As the number of storage increases, however, the degree of precise recollection becomes worse in both models. The performance is slightly better in morphological model though, the difference is not so significant.

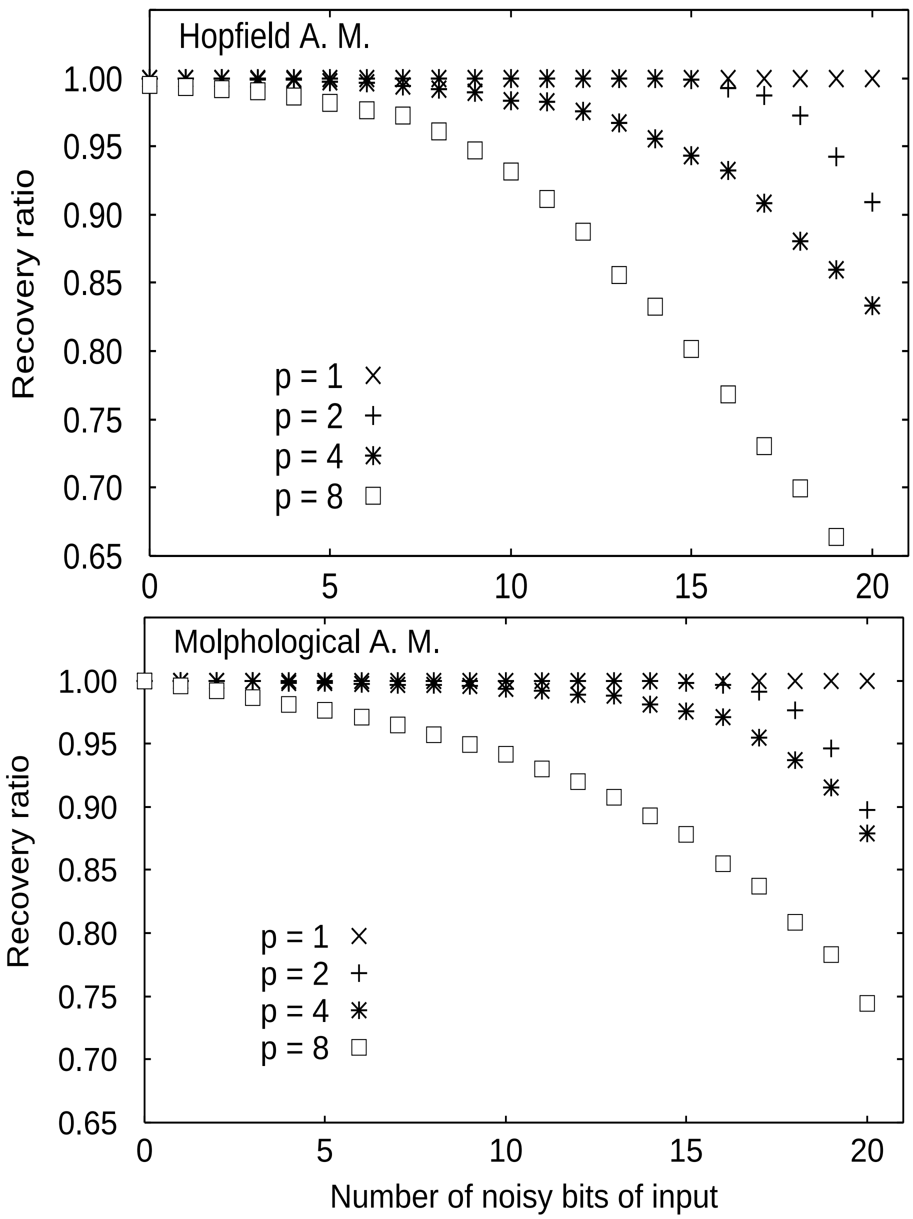

Figure 2: Recovery ratio of recalled patterns vs. number of noisy bits when the number of stored patterns is 1, 2, 4, and 8.

3.3 Basin size dependency on storage

The comparisons are also made for the recovery ratio versus number of storage. As Figure 3 shows, the morphological associative memory also begins to fail to recall a stored pattern as the number of storage increases. However, we found that ratio of correct bit in morphological model is far better than in the Hopfield model. Furthermore, the ratio does not decrease any more after reaching some level of failure, while in the Hopfield model, the ratio keeps decreasing.

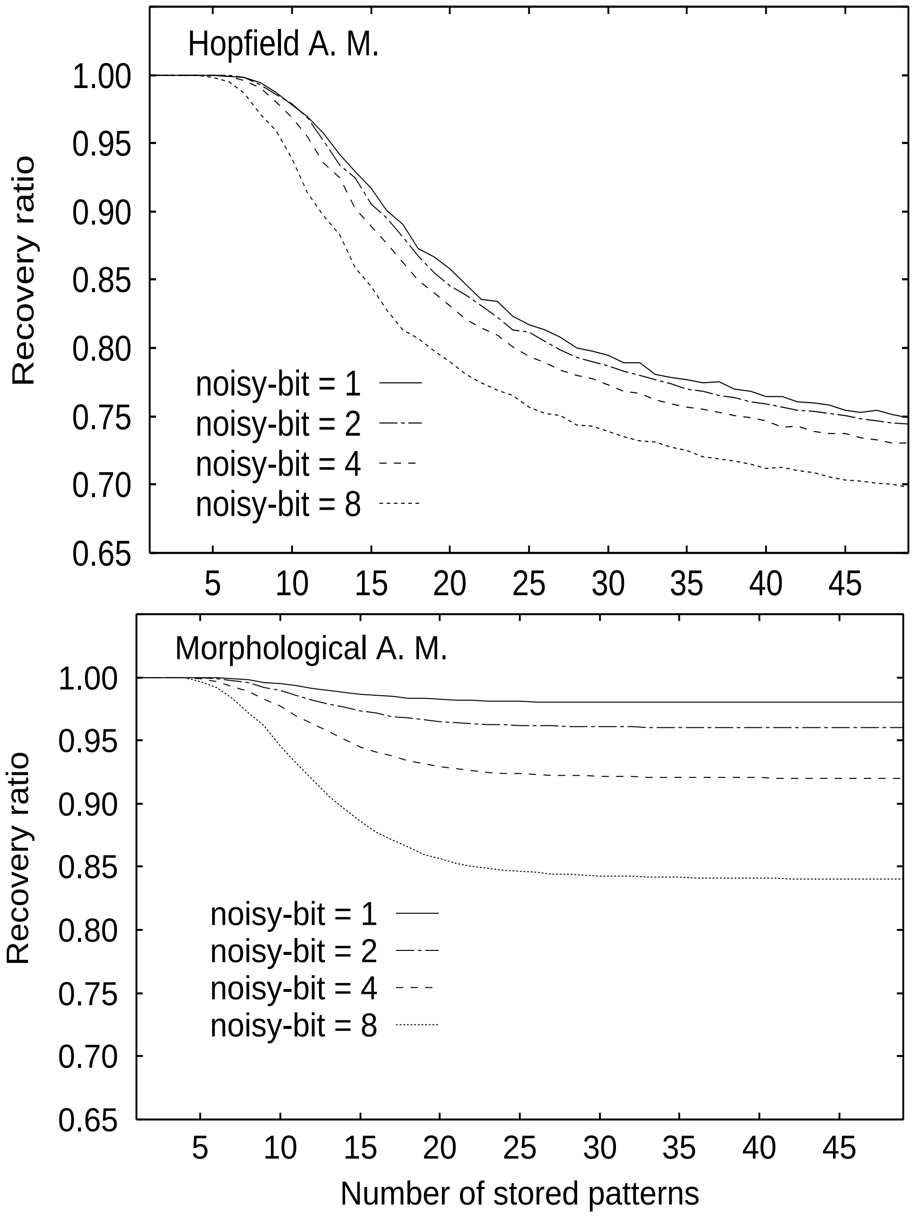

Figure 3: Recovery from 1, 2, 4, and 8 bit noisy input of the stored patterns

4 CONCLUSION

The error correcting capability of the morphological associative memory has empirically studied by comparing it with the Hopfield model. We found that although the morphological associative memory has a large storage capacity as Ritter et al. [?] said in their paper, the error correcting capability, on the other hand, is only slightly better than the one of the Hopfield model. However the robustness of the morphological model for a large number of storage is significantly higher in morphological associative memory than in the Hopfield model.

As Ritter et al. [?] also pointed out, the morphological associative memory model has not been studied so extensively, especially from the view point of error correcting capability. Due to its large storage capacity, the model could be applied more practically to real world problems. For the purpose, however, analysis of error collecting capability is important. Hopefully as a near future work, the behaviors of basin size of attractors will be studied analytically.

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