A-mesa-in-desert vs a-needle-in-haystack:

A fitness landscape on weight space of an application using spiking neurons under rate coding.

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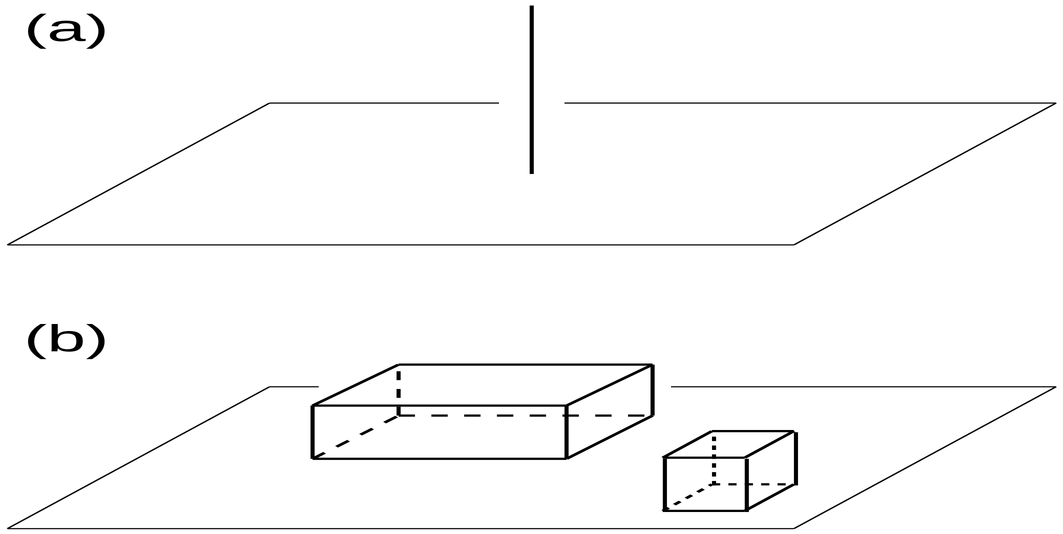
We came accross

an extremely difficult
PEAK
to be searched for

in a
FITNESS LANDSCAPE

explored by an
EVOLUTIONARY COMPUTATION


(a)
(b)

To realize an

ASSOCIATIVE MEMORY

we use

SPIKING NEURONS

whose learning is by

EVOLUTIONARY COMPUTATIONS.

□ Contents

(cont'd)

3. Results of our Experiments.

- **Downhill Walk**
- **Hillclimbing**
- **Hinton & Nowlan's Experiment of Baldwin Effect**
  - **Effect of Lifetime Learning**

4. Summary & Conclusions.

□Associative Memory:

- **stores patterns**
  - **in a distributed way (among neurons),**
- **recalls patterns**
  - **from noisy and/or partial input.**

$\Downarrow$ **i.e.**

- **gives us**
  - **perfect recollection from imperfect information**

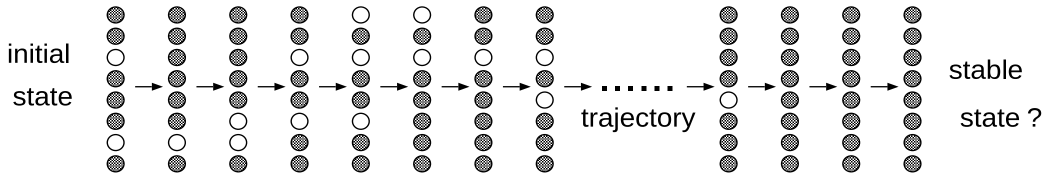
□The Associative Memory can be realized with:

- **Fully-connected Neural Network Model**

  (Hopfield type)
- **Artificial Immune System Model**
- **Spiking Neurons**

  (To be more biologically plausible)
- **etc.**

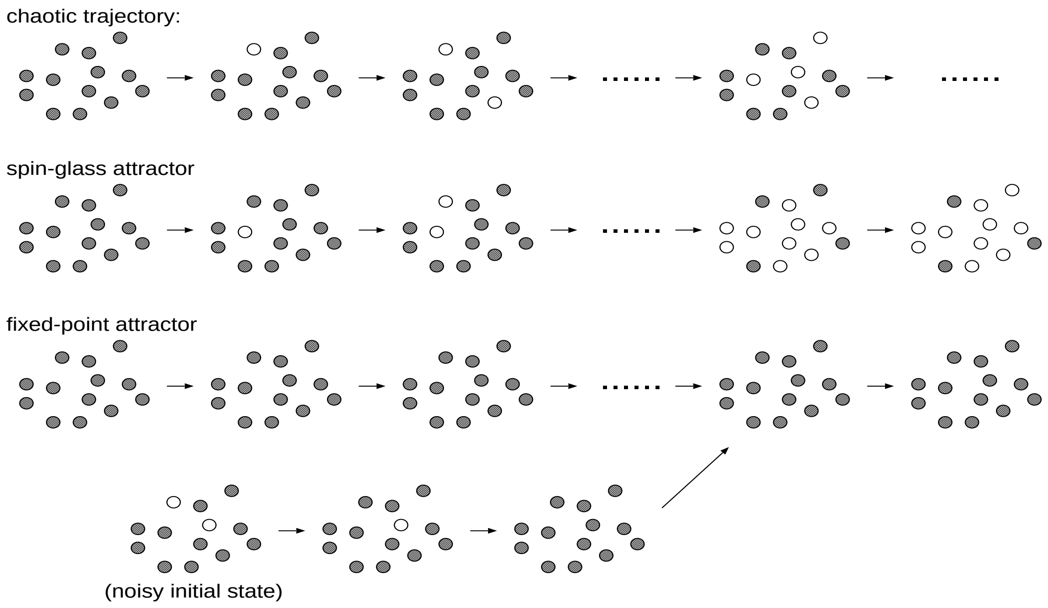
associative memory = dynamical system

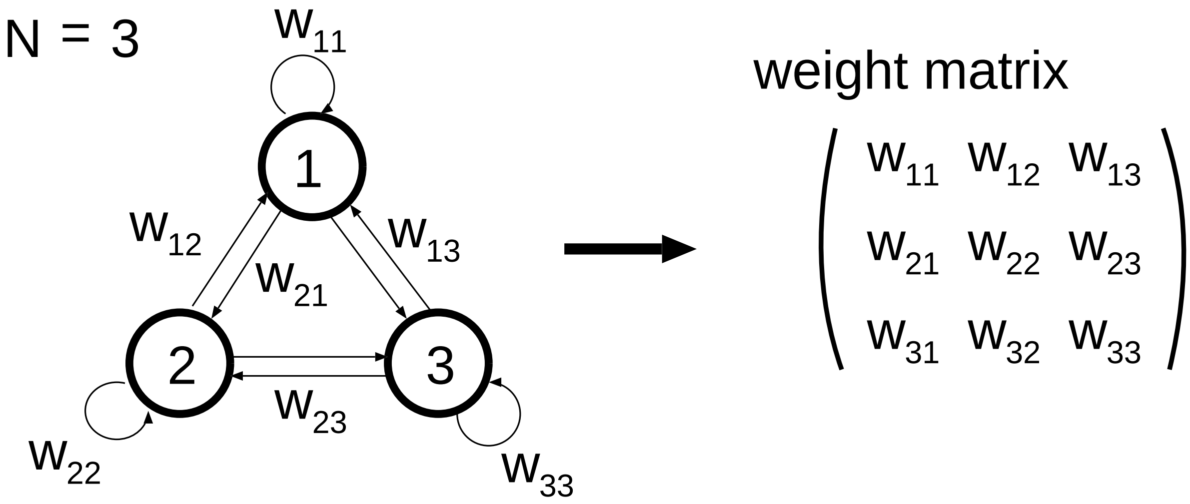


converge

initial state $\xrightarrow{\text{converge}}$ equilibrium state

u+du $\longrightarrow$ u


chaotic trajectory:
......
......
spin-glass attractor
......
fixed-point attractor
......
(noisy initial state)



N = 3
W11
W12
W13
W21
W23
W22
W33
1
2
3
weight matrix
W11 W12 W13
W21 W22 W23
W31 W32 W33

□ State Transition of the Hopfield Model

$$s_i(t+1) = sgn(\sum_j^N w_{ij} s_j(t))$$

□ Hebbian Weights

$$w_{ij} = \frac{1}{N} \sum_{\mu=1}^{p} \xi_i^{\nu} \xi_j^{\nu}, \quad (i \neq j), \quad w_{ij} = 0.$$

(To store p patterns: $\mathbf{x}^{\mu} = (\xi_1^{\mu}, \xi_2^{\mu}, \cdots \xi_N^{\mu}) \quad \mu = 1, 2, \cdots, p$)

□ Hopfield Model has a Crucial Drawback.

- N neurons store only $2N$ patterns at most (Gardner).

$$p < 2N.$$

□ From the Hopfield Model to Spiking Neuron Model

Influence from other neurons
via
ELECTRIC CURRENTS

$\Downarrow$

changes
MEMBRANE VOLTAGE
of the neuron.

(cont'd)

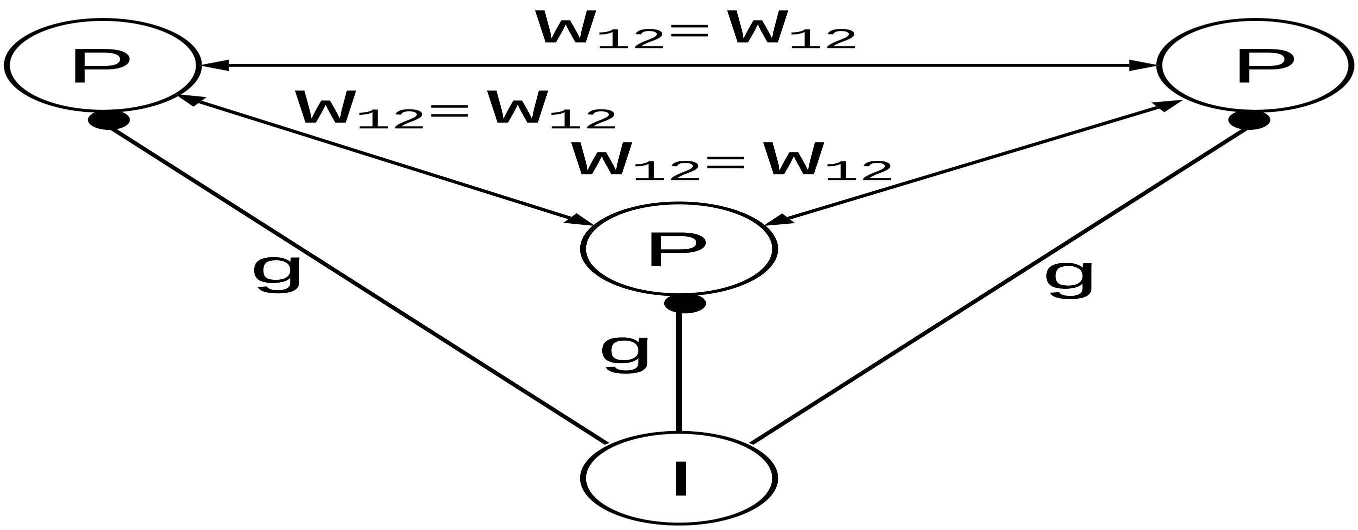
$\Downarrow$

If it exceeds a threshold

$\Downarrow$

the neuron emits
a SPIKE.

□ Pyramidal Cells & Interneurons



PYRAMIDAL CELL

$\|$
(positive current)
$\Downarrow$

PYRAMIDAL CELL

$\Uparrow$
(negative current)
$\|$

INTERNEURON

□ Response of a Single Neuron to an External Stimulus:

- **Spike rate vs stimuli (Naka-Rushton function)**

$$S(P) = \begin{cases} MP^n/(\sigma^n + P^n) & \text{if } P \geq 0 \\ 0 & \text{if } P < 0 \end{cases}$$

(cont'd)

- **Spike rate under time-variant stimulus:**

$$\frac{dr(t)}{dt} = \frac{1}{\tau}(-r(t) + S(P))$$

□ Response of Multiple Neurons

- **Stimulus to a Pyramidal Cell**

$$P_i = (\sum_{j=1}^{N} w_{ij} \cdot R_j - g \cdot G)_+^2$$

of

$$S(P_i) = MP_i^n/(\sigma^n + P_i^n)$$

(cont'd)

- **Spiking Ratio of a Pyramidal Cell**

$$\frac{dr}{dt} = \frac{1}{\tau}(-r + MP^n/(\sigma^n + P^n))$$

becomes

$$\tau_R \frac{dR_i}{dt} = -R_i + \frac{100(\Sigma_{j=1}^{N} w_{ij} R_j - 0.1G)_+^2}{100 + (\Sigma_{j=1}^{N} w_{ij} R_j - 0.1G)_+^2}$$

(cont'd)

- **Spiking Ratio of Interneuron**

$$\tau_G \frac{dG}{dt} = -G - 0.07 \sum_{j=1}^{N} R_i$$

□ How to encode patterns?

$$\xi_i = \begin{cases} 1 & \text{if } R_i \geq M/2 \\ 0 & \text{if } R_i < M/2 \end{cases}$$

$\Rightarrow$ Rate Coding

- Rate Coding
- Time Coding

□ GA Implementation:

1. Represent a series of w_{ij} as a *population* of strings (*chromosome*).

W_{11}	W_{12}	W_{13}	···	W_{21}	·········	W_{N1}	···	W_{NN}

2. Evaluate *fitness* by "How good is each individual?"

3. Generate an initial *population* at random.

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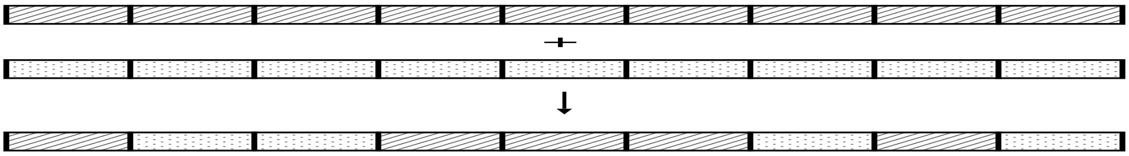
4. Evolve them with
   - *Selection*
   - *Crossover*



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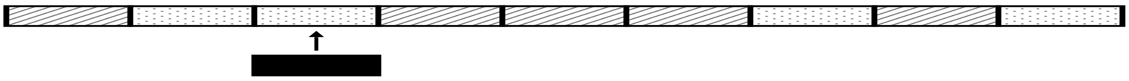
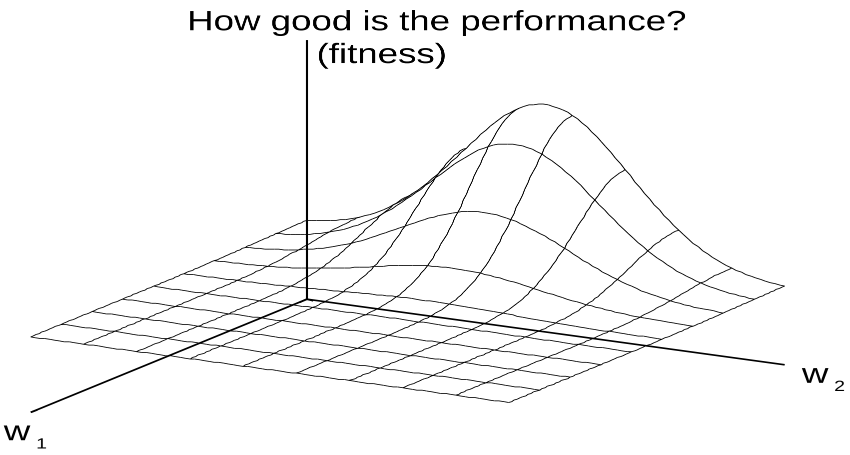
– *Mutation*



5. Better Solutions from *generation* to *generation.*

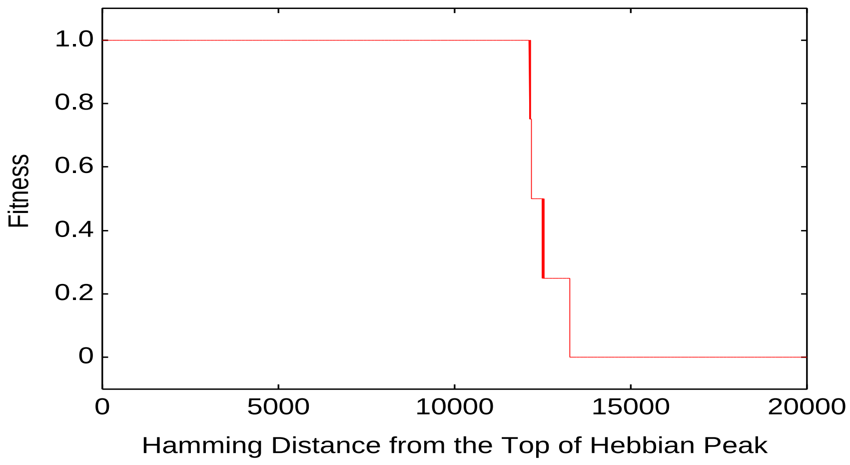
□ A Conceptual Illustration of Fitness Landscape



□ Weights by Hebb's Rule

$$w_{ij} = sgn(R_i - M/2) \cdot sgn(R_j - M/2)$$

- **A Downhill Walk by Flipping zero to one:**



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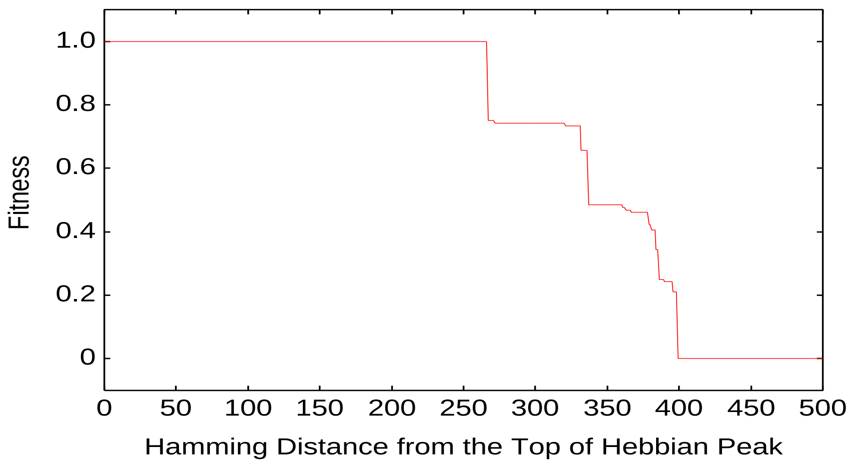
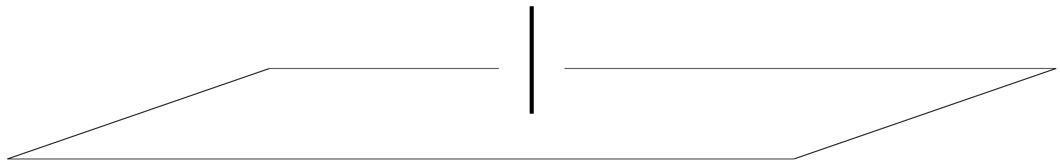
- **A Downhill Walk by Flipping one to zero:**

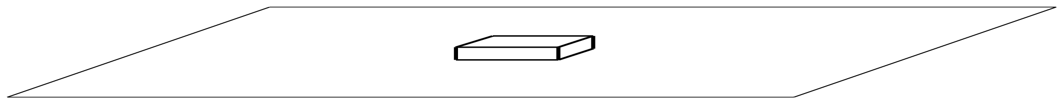


□ Search for a mesa

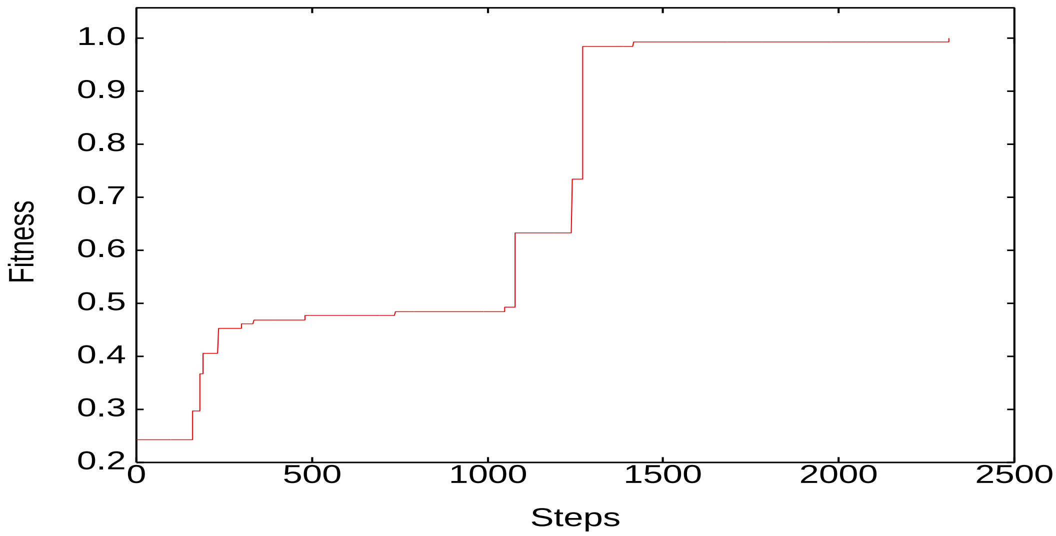
(a) A needle in haystack



(b) A mesa in desert



□ Hill climbing from very edge of sidewall.



□ Hinton & Nolan's Search for A-needle-in-haystack

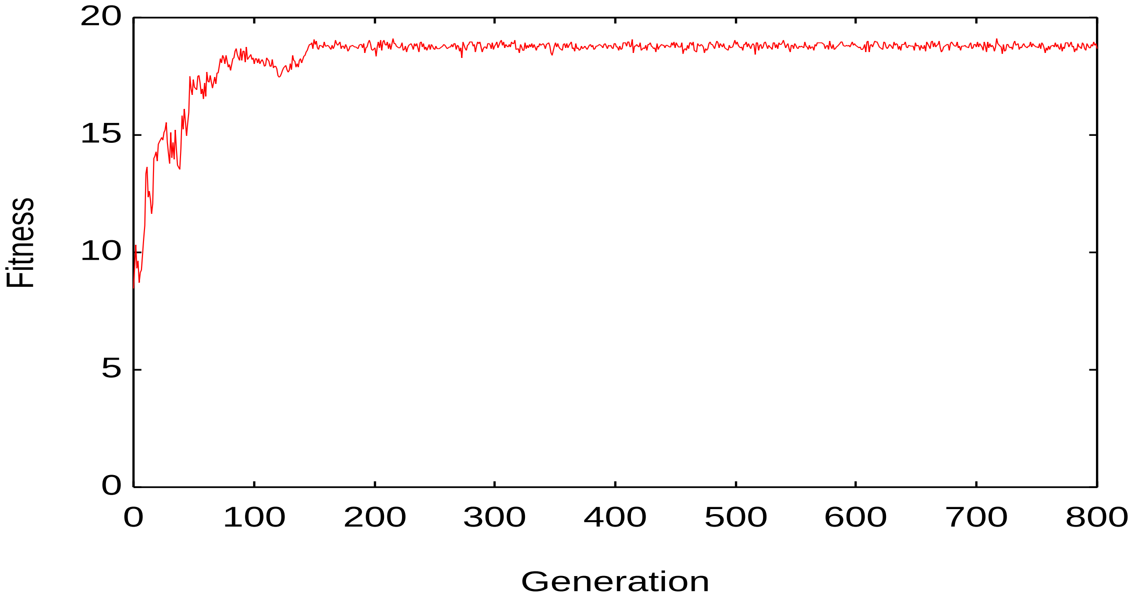
- A-needle ⇒ Only one configuration of 20 bits of binary string.
- Haystack ⇒ 2^{20} search points.
  - · say, (11111111110000000000) is assigned fitness one, while others zero.

(cont'd)

- *lifetime learning* of each individual.
  - about 25% are "1", 25% are "0", and the rest of the 50% are "?" .
  - During an evolution all the "?" position is assigned one or zero at random $\Rightarrow$ learning
  - Repeat the learning upto 1000 times
  - If it reaches the point of fitness one at the n-th trial, then the *degree to which learning succeeded* is calculated as

$$1 + 19 \cdot (1000 - n)/1000.$$

□ Search for a needle



□ Conclusion

- **Search for weights to store patterns is more difficult than a search for**
  - **· a needle in haystack.**

- **We guess it's due to very steep side wall of the peak.**
  - **· A mesa rather than a peak?**

(cont'd)

- **Can climbers take the uphill slope again?**
  - · **Yes, but only from the edge.**

- **Hinton & Nowran's Experiment?**
  - · **Effect of learning during life time (Baldwin Effect).**