On the Inspection of Classification Results in the Fuzzy Clustering Method Based on the Allotment Concept

Dmitri A. Viattchenin

Military Academy of The Republic of Belarus

220057 Minsk, Belarus

e-mail: viattchenin@mail.ru

Abstract: *The paper deals in the preliminary way with the problem of an evaluation of fuzzy clustering results. Basic concepts of the AFC-method of fuzzy clustering are considered and some measures for the evaluation of fuzzy clustering results are proposed. Results of numerical experiments are presented and preliminary conclusions are made.*

***Keywords*:** fuzzy cluster, allotment, linear index of fuzziness, quadratic index of fuzziness, density of fuzzy cluster

I. INTRODUCTION

An outline for a new approach to fuzzy clustering was presented in [1], where a concept of allotment among fuzzy clusters was introduced and a basic version of heuristic fuzzy clustering method was described. The main goal of the paper is a consideration of a problem of inspection of fuzzy clustering results. For this purpose, basic concepts of the allotment among fuzzy clusters (AFC) method are considered. Illustrative examples are shown and conclusions are formulated.

II. BASIC CONCEPTS

Let us consider basic definitions of the AFC-algorithm which are considered in detail in [1].

Definition 1. Let $X=\{x_1,...,x_n\}$ be the initial set of elements and $T: X \times X \to [0,1]$ some binary fuzzy relation on $X=\{x_1,...,x_n\}$ with $\mu_T(x_i,x_j)\in[0,1], \forall x_i,x_j \in X$ being its membership function. The fuzzy binary intransitive relation T which possesses the symmetricity property and the reflexivity property is the fuzzy tolerance relation on X.

Let T be a fuzzy tolerance on X and α be α-level value of T, $\alpha \in (0,1]$.

Definition 2. The α-level fuzzy set $A^l_{(\alpha)}=\{(x_i,\mu_{A^l}(x_i)) \mid \mu_{A^l}(x_i)\geq\alpha, x_i\in X, l\in[1,n]\}$ is fuzzy α-cluster or fuzzy cluster in simple words. So $A^l_{(\alpha)} \subseteq A^l, \alpha\in(0,1], A^l \in \{A^1,\ldots,A^n\}$ and μ_{li} is the membership degree of the element $x_i \in X$ for some fuzzy cluster $A^l_{(\alpha)}, \alpha\in(0,1], l\in[1,n]$. Value of α is the tolerance threshold of fuzzy clusters elements.

The membership degree of the element $x_i \in X$ for some fuzzy cluster $A^l_{(\alpha)}, \alpha\in(0,1], l\in[1,n]$ can be defined as

$$\mu_{li}=\begin{cases}\mu_{A^l}(x_i), & x_i\in A^l_\alpha \\ 0, & else\end{cases}, \tag{1}$$

where $A^l_\alpha=\{x_i\in X \mid \mu_{A^l}(x_i)\geq\alpha\}, \alpha\in(0,1]$ is the support of the fuzzy cluster $A^l_{(\alpha)}$. A condition $A^l_\alpha = Supp(A^l_{(\alpha)})$ is met for each fuzzy cluster $A^l_{(\alpha)}, \alpha\in(0,1], l=\overline{1,n}$.

Definition 3. If T is a fuzzy tolerance on X, where X is the set of elements, and $\{A^1_{(\alpha)},...,A^n_{(\alpha)}\}$ is the family of fuzzy clusters for some $\alpha\in(0,1]$, then the point $\tau^l_e \in A^l_\alpha$, for which

$$\tau^l_e = \arg\max_{x_i} \mu_{li}, \forall x_i \in A^l_\alpha \tag{2}$$

is called a typical point of the fuzzy cluster $A^l_{(\alpha)}, \alpha\in(0,1], l\in[1,n]$.

Definition 4. Let $R^\alpha_z(X)=\{A^l_{(\alpha)} \mid l=\overline{1,c}, 2\leq c\leq n\}$ be a family of fuzzy clusters for some value of tolerance threshold $\alpha, \alpha\in(0,1]$, which are generated by some fuzzy tolerance T on the initial set of elements $X=\{x_1,...,x_n\}$. If a condition

$$\sum_{l=1}^{c}\mu_{li} > 0, \forall x_i \in X \tag{3}$$

is met for all $A^l_{(\alpha)}, l=\overline{1,c}, c\leq n$, then the family is the allotment of elements of the set $X=\{x_1,...,x_n\}$ among fuzzy clusters $\{A^l_{(\alpha)}, l=\overline{1,c}, 2\leq c\leq n\}$ for some value of tolerance threshold $\alpha, \alpha\in(0,1]$.

If some allotment $R^\alpha_z(X)=\{A^l_{(\alpha)} \mid l=\overline{1,c}, c\leq n\}$ corresponds to the formulation of a concrete problem, then this allotment is an adequate allotment. In particular, if a condition

$$\bigcup_{l=1}^{c} A^l_\alpha = X, \tag{4}$$

and a condition

$$card(A_\alpha^l \cap A_\alpha^m) = 0, \forall A_{(\alpha)}^l, A_{(\alpha)}^m, l \neq m, \alpha \in (0,1] \quad (5)$$

are met for all fuzzy clusters $A_{(\alpha)}^l, l = \overline{1,c}$ of some allotment $R_z^\alpha(X) = \{A_{(\alpha)}^l \mid l = \overline{1,c}, c \leq n, \alpha \in (0,1]\}$ then the allotment is the allotment among fully separate fuzzy clusters. However, fuzzy clusters in the sense of definition 2 can have an intersection area. So, the conditions (4) and (5) can be generalized [2]. In particular, a condition

$$\begin{gathered}\sum_{l=1}^{c} card(A_\alpha^l) \geq card(X), \\ \forall A_{(\alpha)}^l \in R_z^\alpha(X), card(R_z^\alpha(X)) = c\end{gathered}, \quad (6)$$

and a condition

$$\begin{gathered}card(A_\alpha^l \cap A_\alpha^m) \leq w, \\ \forall A_{(\alpha)}^l, A_{(\alpha)}^m, l \neq m\end{gathered}, \quad (7)$$

are generalization of the conditions (4) and (5).

So, the problem of discovering the unique allotment $R^*(X)$ of the initial set of elements $X = \{x_1, ..., x_n\}$ among c fully separate or particularly separate fuzzy clusters is the problem of classification. The matrix of similarity coefficients $T = [\mu_T(x_i, x_j)], i, j = 1, \ldots, n$ is the matrix of initial data for the AFC-algorithm. The allotment $R^*(X)$ among c fuzzy clusters and tolerance threshold α are results of the classification process. The general plan of the AFC-algorithm for the problem solving is presented in [1].

III. EVALUATION OF FUZZY CLUSTERS

The qualitative inspection of fuzzy clustering results can be done, e.g., with a linear index of fuzziness or a quadratic index of fuzziness, used for evaluation of fuzziness degree of fuzzy clusters. These two indexes are introduced and considered in [3].

The linear index of fuzziness is defined as

$$I_L(A_{(\alpha)}^l) = \frac{2}{n_l} \cdot d_H(A_{(\alpha)}^l, \underline{A_{(\alpha)}^l}), \quad (8)$$

where $n_l = card(A_\alpha^l), A_{(\alpha)}^l \in R^*(X)$ is the number of objects in the fuzzy cluster $A_{(\alpha)}^l$ and $d_H(A_{(\alpha)}^l, \underline{A_{(\alpha)}^l})$ is the Hamming distance

$$d_H(A_{(\alpha)}^l, \underline{A_{(\alpha)}^l}) = \sum_{x_i \in A_\alpha^l} \left| \mu_{li} - \mu_{\underline{A_{(\alpha)}^l}}(x_i) \right| \quad (9)$$

between the fuzzy cluster $A_{(\alpha)}^l$ and the crisp set $\underline{A_{(\alpha)}^l}$ nearest to the fuzzy cluster $A_{(\alpha)}^l$. The membership function of the crisp set $\underline{A_{(\alpha)}^l}$ can be defined as

$$\mu_{\underline{A_{(\alpha)}^l}}(x_i) = \begin{cases} 0, \mu_{A_{(\alpha)}^l}(x_i) \leq 0.5 \\ 1, \mu_{A_{(\alpha)}^l}(x_i) > 0.5 \end{cases}, \forall x_i \in A_\alpha^l, \quad (10)$$

where $\alpha \in (0,1]$.

The quadratic index of fuzziness is defined as

$$I_Q(A_{(\alpha)}^l) = \frac{2}{\sqrt{n_l}} \cdot d_E(A_{(\alpha)}^l, \underline{A_{(\alpha)}^l}), \quad (11)$$

where $n_l = card(A_\alpha^l), A_{(\alpha)}^l \in R^*(X)$ and $d_E(A_{(\alpha)}^l, \underline{A_{(\alpha)}^l})$ is the Euclidean distance

$$d_E(A_{(\alpha)}^l, \underline{A_{(\alpha)}^l}) = \sqrt{\sum_{x_i \in A_\alpha^l} \left(\mu_{li} - \mu_{\underline{A_{(\alpha)}^l}}(x_i) \right)^2} \quad (12)$$

between the fuzzy cluster $A_{(\alpha)}^l$ and the crisp set $\underline{A_{(\alpha)}^l}$ which are defined by formula (10). For each fuzzy cluster $A_{(\alpha)}^l$ in $R^*(X)$, evidently, the following conditions are met:

$$0 \leq I_L(A_{(\alpha)}^l) \leq 1, \quad (13)$$

$$0 \leq I_Q(A_{(\alpha)}^l) \leq 1. \quad (14)$$

Indexes (8) and (11) show the degree of fuzziness of bounds of fuzzy clusters which are elements of the allotment $R^*(X)$. Obviously, that $I_L(A_{(\alpha)}^l) = I_Q(A_{(\alpha)}^l) = 0$ for a crisp set $A_{(\alpha)}^l \in R^*(X)$. Otherwise, if $\mu_{li} = 0.5, \forall x_i \in A_\alpha^l$ then fuzzy cluster $A_{(\alpha)}^l \in R^*(X)$ is a maximal fuzzy set and $I_L(A_{(\alpha)}^l) = I_Q(A_{(\alpha)}^l) = 1$.

A density of fuzzy cluster can be defined as

$$D(A_{(\alpha)}^l) = \frac{1}{n_l} \sum_{x_i \in A_\alpha^l} \mu_{li}, \quad (15)$$

where $n_l = card(A_\alpha^l), A_{(\alpha)}^l \in R^*(X)$ and membership degree μ_{li} is defined by formula (1). It is obvious, that a condition

$$0 \leq D(A_{(\alpha)}^l) \leq 1, \quad (16)$$

is met for each fuzzy cluster $A_{(\alpha)}^l$ in $R^*(X)$. Moreover, $D(A_{(\alpha)}^l) = 1$ for a crisp set $A_{(\alpha)}^l \in R^*(X)$ for any tolerance threshold $\alpha, \alpha \in (0,1]$. The density of fuzzy cluster shows an average membership degree of elements of a fuzzy cluster.

IV. EXPERIMENTAL RESULTS

The Anderson's Iris data [4] consist of sepal length, sepal width, petal length and petal width for 150 irises. The problem is to classify the plants into three subspecies on the basis of this information. The real assignments to the three classes are shown in Table 1.

Table 1. Real objects assignment

Class		Numbers of objects
Number	**Name**	
1	SETOSA	1, 6, 10, 18, 26, 31, 36, 37, 40, 42, 44, 47, 50, 51, 53, 54, 55, 58, 59, 60, 63, 64, 67, 68, 71, 72, 78, 79, 87, 88, 91, 95, 96, 100, 101, 106, 107, 112, 115, 124, 125, 134, 135, 136, 138, 139, 143, 144, 145, 149
2	VERSICOLOR	3, 8, 9, 11, 12, 14, 19, 22, 28, 29, 30, 33, 38, 43, 48, 61, 65, 66, 69, 70, 76, 84, 85, 86, 92, 93, 94, 97, 98, 99, 103, 105, 109, 113, 114, 116, 117, 118, 119, 120, 121, 128, 129, 130, 133, 140, 141, 142, 147, 150
3	VIRGINICA	2, 4, 5, 7, 13, 15, 16, 17, 20, 21, 23, 24, 25, 27, 32, 34, 35, 39, 41, 45, 46, 49, 52, 56, 57, 62, 73, 74, 75, 77, 80, 81, 82, 83, 89, 90, 102, 104, 108, 110, 111, 122, 123, 126, 127, 131, 132, 137, 146, 148

The matrix of attributes is the matrix $X_{m \times n} = [x_i^t], i = 1, \ldots, n, t = 1, \ldots, m$, where $n = 150, m = 4$. So, the value x_i^t is the value of t-th attribute for i-th object. The data can be normalized as follows:

$$\mu_{x_i}(x^t) = \frac{x_i^t}{\max\limits_{x_i} x_i^t}, i = 1, \ldots, n, \qquad (17)$$

for all attributes $x^t, t = 1, \ldots, m$. So, each object can be considered as a fuzzy set $x_i, i = 1, \ldots, n$ and $\mu_{x_i}(x^t) \in [0,1], i = 1, \ldots, n, t = 1, \ldots, m$ are their membership functions. After application of a distance $d(x_i, x_j), i, j = \overline{1, n}$ to the matrix of normalized data $X'_{m \times n} = [\mu_{x_i}(x^t)], i = 1, \ldots, n, t = 1, \ldots, m$ a matrix of fuzzy intolerance $I = [\mu_I(x_i, x_j)], i, j = 1, \ldots, n$ can be obtained. The matrix of fuzzy tolerance $T = [\mu_T(x_i, x_j)], i, j = 1, \ldots, n$ can be obtained after application of complement operation

$$\mu_T(x_i, x_j) = 1 - \mu_I(x_i, x_j), \forall i, j = 1, \ldots, n \qquad (18)$$

to the matrix of fuzzy intolerance $I = [\mu_I(x_i, x_j)], i, j = 1, \ldots, n$.

A few distances can be used as the $d(x_i, x_j), i, j = \overline{1, n}$ distance. The most widely used distances for any two fuzzy sets x_i, x_j in $X = \{x_1, ..., x_n\}$ are [3]:

- The normalized Hamming distance:

$$l(x_i, x_j) = \frac{1}{m} \sum_{t=1}^{m} | \mu_{x_i}(x^t) - \mu_{x_j}(x^t) |, \quad i, j = \overline{1, n}, \qquad (19)$$

- The normalized Euclidean distance:

$$e(x_i, x_j) = \sqrt{\frac{1}{m} \sum_{t=1}^{m} \left(\mu_{x_i}(x^t) - \mu_{x_j}(x^t) \right)^2}, \quad i, j = \overline{1, n}, \qquad (20)$$

- The squared normalized Euclidean distance:

$$\varepsilon(x_i, x_j) = \frac{1}{m} \sum_{t=1}^{m} \left(\mu_{x_i}(x^t) - \mu_{x_j}(x^t) \right)^2, \quad i, j = \overline{1, n}. \qquad (21)$$

In the case of the normalized Hamming distance the allotment $R^*(X)$ which corresponds to the result, was obtained for the tolerance threshold $\alpha = 0.8192$. Fourteen mistakes of classification were received. Supports of fuzzy clusters in the case of the normalized Hamming distance are presented in Table 2. Misclassified objects are distinguished in Table 2.

Table 2. The objects assignment in the case of the normalized Hamming distance

Class		Numbers of objects
Number	**Name**	
1	SETOSA	1, 6, 10, 18, 26, 31, 36, 37, 40, 42, 44, 47, 50, 51, 53, 54, 55, 58, 59, 60, 63, 64, 67, 68, 71, 72, 78, 79, 87, 88, 91, 95, 96, 100, 101, 106, 107, 112, 115, 124, 125, 134, 135, 136, 138, 139, 143, 144, 145, 149
2	VERSICOLOR	3, **5**, 8, 9, 11, 12, 14, **16**, 19, 22, **25**, 28, 29, 30, **32**, 33, **34**, 38, 43, **46**, 48, **52**, **56**, 61, **62**, 65, 66, 69, 70, **75**, 76, **82**, 84, 85, 86, **90**, 92, 93, 94, 97, 98, 99, 103, 105, **108**, 109, 113, 114, 116, 117, 118, 119, 120, 121, 128, 129, 130, 133, **137**, 140, 141, 142, 150
		2, 4, 7, 13, 15, 16, 17, 20, 21, 23, 24, 27, 35, 39, 41,

3	VIRGINICA	45, 49, 57, 73, 74, 77, 80, 81, 83, 89, 102, 104, 110, 111, 122, 123, 126, 127, 131, 132, 137, 146, **147**, 148

The object x_{95} is the typical point τ^1 of the fuzzy cluster which corresponds to the first class. The object x_{98} is the typical point τ^2 of the fuzzy cluster which corresponds to the second class and the object x_{126} is the typical point τ^3 of the fuzzy cluster which corresponds to the third class. Membership values of the Setosa class are presented in Fig. 1.

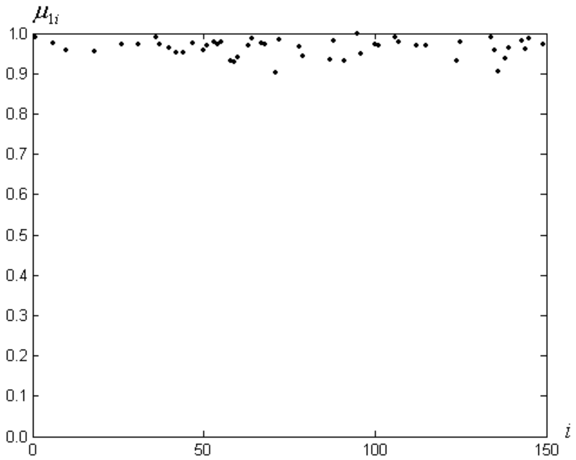

Fig. 1. – Membership values of the SETOSA class in the case of the normalized Hamming distance

Membership values of the Versicolor class are presented in Fig. 2.

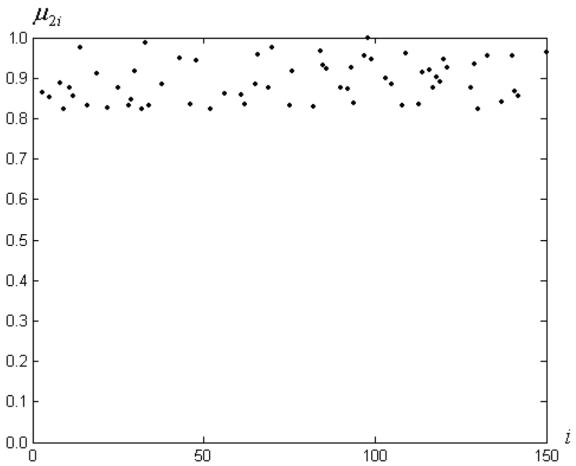

Fig. 2. – Membership values of the VERSICOLOR class in the case of the normalized Hamming distance

Membership values of the Virginica class are presented in Fig. 3.

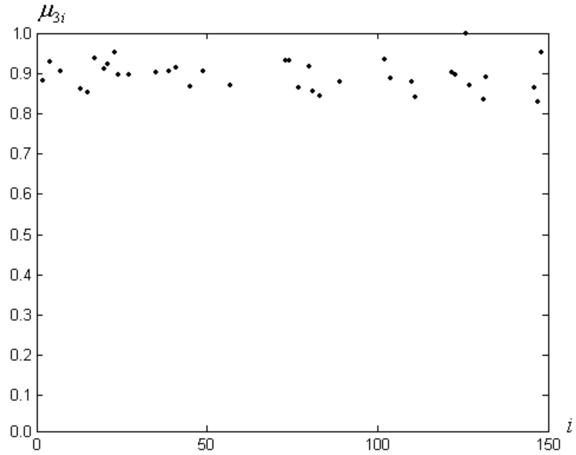

Fig. 3. – Membership values of the VIRGINICA class in the case of the normalized Hamming distance

Values of the linear index of fuzziness, the quadratic index and the density of fuzzy clusters are presented in Table 3.

Table 3. Results of the evaluation of fuzzy clusters in the case of the normalized Hamming distance

Numbers of classes	The value of		
	the linear index of fuzziness	the quadratic index of fuzziness	the density of fuzzy cluster
1	0.07199	0.08409	0.96400
2	0.21713	0.23934	0.89143
3	0.21044	0.22273	0.89478

In the case of the normalized Euclidean distance the allotment $R^*(X)$ which corresponds to the result, was obtained for the tolerance threshold $\alpha = 0.8104$. Six mistakes of classification were received. The object x_{95} is the typical point τ^1 of the fuzzy cluster which corresponds to the first class. The object x_{98} is the typical point τ^2 of the fuzzy cluster which corresponds to the second class and the object x_{23} is the typical point τ^3 of the fuzzy cluster which corresponds to the third class. Supports of fuzzy clusters in the case of the normalized Euclidean distance are presented in Table 4. Misclassified objects are distinguished in Table 4.

Table 4. The objects assignment in the case of the normalized Euclidean distance

Class		Numbers of objects
Number	Name	
1	SETOSA	1, 6, 10, 18, 26, 31, 36, 37, 40, 42, 44, 47, 50, 51, 53, 54, 55, 58, 59, 60, 63, 64, 67, 68, 71, 72, 78, 79, 87, 88, 91, 95, 96, 100, 101, 106, 107, 112, 115, 124, 125, 134, 135, 136, 138, 139, 143, 144, 145, 149
		3, **5**, 8, 11, 12, 14, 19, 22,

2	VERSICOLOR	**25**, 28, 29, 30, 33, 38, 43, 48, **56**, 61, 65, 66, 69, 70, 76, 84, 85, 86, **90**, 92, 93, 94, 97, 98, 99, 103, 105, 109, 113, 114, 116, 117, 118, 119, 120, 121, 128, 129, 130, 133, 140, 141, 142, 150
3	VIRGINICA	2, 4, 7, **9**, 13, 15, 16, 17, 20, 21, 23, 24, 27, 32, 34, 35, 39, 41, 45, 46, 49, 52, 57, 62, 73, 74, 75, 77, 80, 81, 82, 83, 89, 102, 104, 108, 110, 111, 122, 123, 126, 127, 131, 132, 137, 146, **147**, 148

Membership values of the Setosa class are presented in Fig. 4.

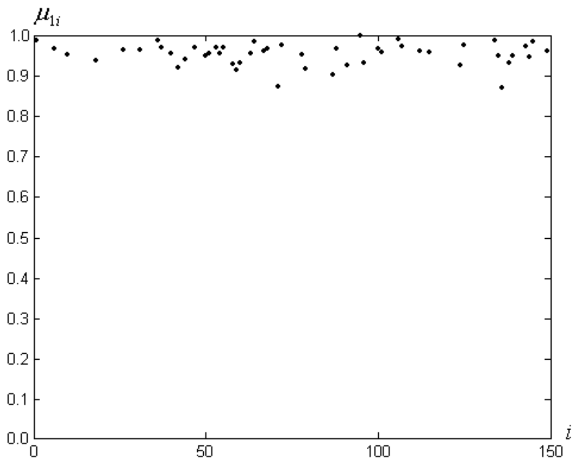

Fig. 4. – Membership values of the SETOSA class in the case of the normalized Euclidean distance

Membership values of the Versicolor class are presented in Fig. 5.

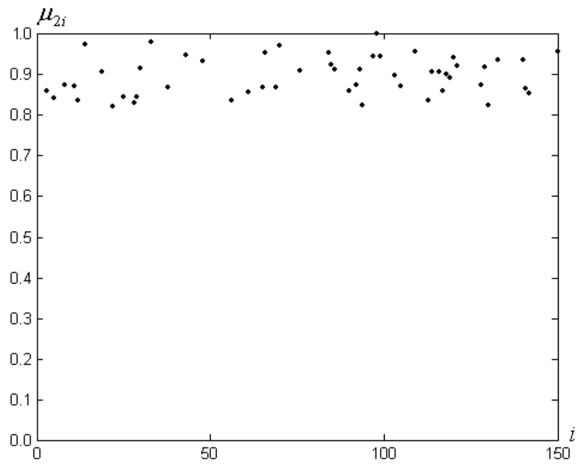

Fig. 5. – Membership values of the VERSICOLOR class in the case of the normalized Euclidean distance

Membership values of the Virginica class are presented in Fig. 6.

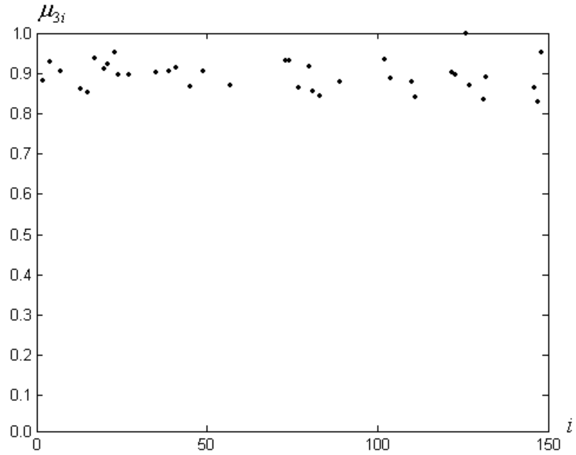

Fig. 6. – Membership values of the VIRGINICA class in the case of the normalized Euclidean distance

Values of the linear index of fuzziness, the quadratic index and the density of fuzzy clusters are presented in Table 5.

Table 5. Results of the evaluation of fuzzy clusters in the case of the normalized Euclidean distance

Numbers of classes	**The value of**		
	the linear index of fuzziness	**the quadratic index of fuzziness**	**the density of fuzzy cluster**
1	0.09497	0.10955	0.95251
2	0.21005	0.22987	0.89497
3	0.22848	0.24782	0.88576

In the case of the normalized Euclidean distance the allotment $R^*(X)$ which corresponds to the result, was obtained for the tolerance threshold $\alpha = 0.9642$. Six mistakes of classification were received. The object x_{95} is the typical point τ^1 of the fuzzy cluster which corresponds to the first class. The object x_{98} is the typical point τ^2 of the fuzzy cluster which corresponds to the second class and the object x_{73} is the typical point τ^3 of the fuzzy cluster which corresponds to the third class. Values of the membership function of typical points of fuzzy clusters are equal one. The object assignments resulting in the case of the normalized Euclidean distance (21) for the Anderson's Iris data preprocessing are presented in Table 6. Misclassified objects are distinguished in Table 6.

Table 6. The objects assignment in the case of the squared normalized Euclidean distance

Class		**Numbers of objects**
Number	**Name**	
1	SETOSA	1, 6, 10, 18, 26, 31, 36, 37, 40, 42, 44, 47, 50, 51, 53, 54, 55, 58, 59, 60, 63, 64, 67, 68, 71, 72, 78, 79, 87, 88, 91, 95, 96, 100, 101,

		106, 107, 112, 115, 124, 125, 134, 135, 136, 138, 139, 143, 144, 145, 149
2	VERSICOLOR	3, **5**, 8, 11, 12, 14, 19, 22, **25**, 28, 29, 30, 33, 38, 43, 48, **56**, 61, 65, 66, 69, 70, 76, 84, 85, 86, **90**, 92, 93, 94, 97, 98, 99, 103, 105, 109, 113, 114, 116, 117, 118, 119, 120, 121, 128, 129, 130, 133, 140, 141, 142, 150
3	VIRGINICA	2, 4, 7, **9**, 13, 15, 16, 17, 20, 21, 23, 24, 27, 32, 34, 35, 39, 41, 45, 46, 49, 52, 57, 62, 73, 74, 75, 77, 80, 81, 82, 83, 89, 102, 104, 108, 110, 111, 122, 123, 126, 127, 131, 132, 137, 146, **147**, 148

Membership values of the Setosa class are presented in Fig. 7.

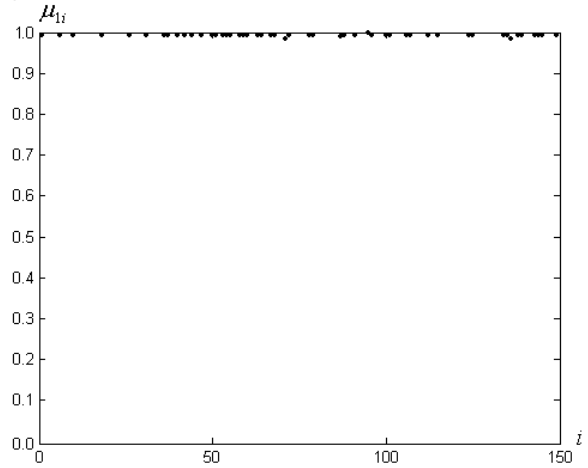

Fig. 7. – Membership values of the SETOSA class in the case of the squared normalized Euclidean distance

Membership values of the Versicolor class are presented in Fig. 8.

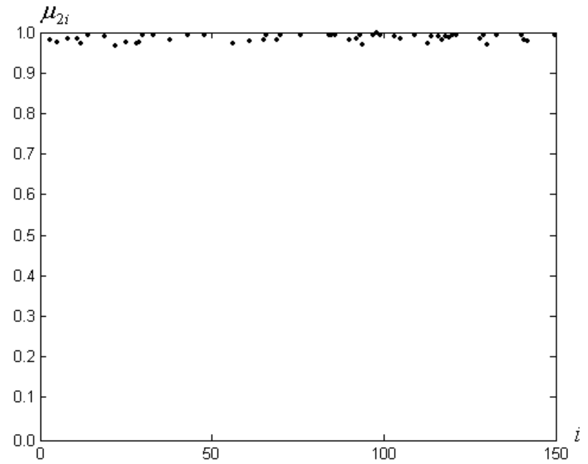

Fig. 8. – Membership values of the VERSICOLOR class in the case of the squared normalized Euclidean distance

Membership values of the Virginica class are presented in Fig. 9.

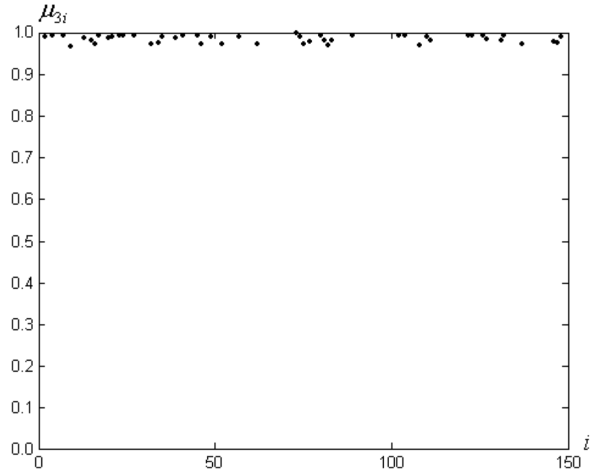

Fig. 9. – Membership values of the VIRGINICA class in the case of the squared normalized Euclidean distance

Values of the linear index of fuzziness, the quadratic index and the density of fuzzy clusters in the case of the squared normalized Euclidean distance for the data preprocessing are presented in Table 7.

Table 7. Results of the evaluation of fuzzy clusters in the case of the squared normalized Euclidean distance

Numbers of classes	**The value of**		
	the linear index of fuzziness	**the quadratic index of fuzziness**	**the density of fuzzy cluster**
1	0.00600	0.00929	0.99700
2	0.02641	0.03265	0.98679
3	0.02942	0.03451	0.98529

If the linear index of fuzziness and the quadratic index of fuzziness are small for some fuzzy cluster, then a shape of the pattern of the fuzzy cluster is crisp. From other hand, if the density of fuzzy cluster is high for some fuzzy cluster, then membership values of elements of fuzzy cluster are high also. So, fuzzy clusters of the allotment $R^*(X)$ which received in the case of the squared normalized Euclidean distance using in data preprocessing are most compact and well-separated fuzzy clusters. Notable, that the squared normalized Euclidean distance is not metric [3].

V. CONCLUSIONS

The results of the allotment method of fuzzy clustering application can be very well interpreted. The allotment method of fuzzy clustering is very simple from the heuristic point of view. Moreover, the objective function-based fuzzy clustering algorithms are sensitive to initialization. Very often, the algorithms are initialized randomly multiple times, in the hope that one of the initializations leads to good clustering results. From other hand, the AFC-algorithm clustering results are stable.

The results of application of the AFC-algorithm to Anderson’s Iris data show that the allotment method of fuzzy clustering is a precise and effective numerical procedure for solving classification problems. Moreover, the linear index of fuzziness, the quadratic index of fuzziness and the density of fuzzy clusters are effectual tools for the inspection of results of the AFC-method application to solving of fuzzy clustering problems.

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