Choice of Free Arguments in Decomposition of Boolean Functions Using the Ternary Matrix Cover Approach

Abstract

The problem of sequential two-block decomposition of completely specified Boolean functions is considered. A method to search for a partition of the set of arguments into the subset of bound arguments and the subset of free ones is suggested. The method is based on using the ternary matrix cover approach.

1 INTRODUCTION

The problem of decomposition of Boolean functions is one of important and complicated problems in the field of logical design, and its successful solution has a direct influence on the quality and cost of digital devices designed. Therefore the search for new efficient methods attracts many researchers. The review (Perkowski, 1995) shows many papers on this topic. We consider the problem of decomposition of a system of Boolean functions in the following statement. A system of completely specified Boolean functions $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})$ is given where $\boldsymbol{y} = (y_1, y_2, \ldots, y_m)$, $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$, $\boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_m(\boldsymbol{x}))$. The superposition $\boldsymbol{y} = \varphi(\boldsymbol{w}, \boldsymbol{z}_2)$, $\boldsymbol{w} = g(\boldsymbol{z}_1)$ where $\boldsymbol{z}_1$ and $\boldsymbol{z}_2$ are vector variables whose components are Boolean variables in the subsets Z_1 and Z_2 respectively that form a partition of the set $X = \{x_1, x_2, \ldots, x_n\}$ of arguments. At that, the number of components of the vector variable $\boldsymbol{w}$ must be less than that of $\boldsymbol{z}_1$. Such a kind of decomposition is called two-block disjoint decomposition by (Zakrevskij, 2009). Usually, the subsets Z_1 and Z_2 are called bound and free sets respectively (bound variables and free ones). In overwhelming majority of publications on the problem, subsets Z_1 and Z_2 are considered to be given. Few papers deal with the search for the partition $\{Z_1, Z_2\}$, at which this problem has a solution**.** Among the papers considering this question, we can point out (Bibilo, 2009; Jóźwiak, 2000; Perkowski, 1995; *Zakrevskij,* 2007). Below, we suggest a method to search a partition $\{Z_1, Z_2\}$ of the set X of arguments based on using the ternary matrix cover technique (*Pottosin, 2006). For this, the concept of compact table is also involve*d that as well as the Karnaugh map is a two-dimensional table, but it represents both a single function and a system of functions in a more compact form. Using a compact table one can find rather easily the existence of a solution of the problem for a given system of functions, and if it does exist, the corresponding superposition can be easily found.

2 MAIN DEFINITIONS. SETTING THE PROBLEM

Let a system of completely specified functions $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})$, where $\boldsymbol{y} = (y_1, y_2, \ldots, y_m)$, $\boldsymbol{x} = (x_1, x_2, \ldots, x_n)$ and $\boldsymbol{f}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \ldots, f_m(\boldsymbol{x}))$, be given by matrices $\boldsymbol{U}$, $\boldsymbol{V}$ that are the matrix representation of the system of disjunctive normal forms (DNFs) of the given functions (Zakrevskij, 2009). Matrix $\boldsymbol{U}$ is a ternary matrix of $l \times n$ dimension where l is the number of terms in the given DNFs. The columns of $\boldsymbol{U}$ are marked with the variables $x_1, x_2, \ldots, x_n$, and the rows represent the terms of the DNFs (the intervals of the space of the variables $x_1, x_2, \ldots, x_n$). The matrix $\boldsymbol{V}$ is a Boolean matrix. Its dimension is $l \times m$, and шеф columns are marked with the variables $y_1, y_2, \ldots, y_m$. The ones in this columns point out the terms in the given DNFs. A row $\boldsymbol{u}$ in $\boldsymbol{U}$ *absorbs* a Boolean vector $\boldsymbol{a}$ if $\boldsymbol{a}$ belongs to the interval represented by $\boldsymbol{u}$.

The task considered is set as follows. Given a system of completely specified Boolean functions $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})$, the superposition $\boldsymbol{y} = \varphi(\boldsymbol{w}, \boldsymbol{z}_2)$, $\boldsymbol{w} = g(\boldsymbol{z}_1)$ must be found where $\boldsymbol{z}_1$ and $\boldsymbol{z}_2$ are vector variables whose components are Boolean variables in the subsets of the set $X = \{x_1, x_2, \ldots, x_n\}$, Z_1 and Z_2 respectively such that $X = Z_1 \cup Z_2$ and $Z_1 \cap Z_2 = \varnothing$. At that, the number of components of the vector variable $\boldsymbol{w}$ must be less than that of $\boldsymbol{z}_1$. The main attention is paid to the search for subsets Z_1 and Z_2 such that the task would have a solution.

3 COVER MAP. COMPACT TABLE

Any family π of different subsets of a set L whose union is L, is called a *cover* of L. Empty set $\varnothing$ and L itself may be the elements (blocks) of π. Let $L = \{1, 2, \ldots, l\}$ be the set of numbers of rows of a ternary matrix $\boldsymbol{U}$. A cover π of L is called a *cover of the ternary matrix* $\boldsymbol{U}$ if for each value $\boldsymbol{x}^*$ of the vector variable $\boldsymbol{x}$ there exists a block in π containing all the numbers of those and only those rows of $\boldsymbol{U}$, which absorb $\boldsymbol{x}^*$. Block $\varnothing$ corresponds to the value $\boldsymbol{x}^*$, which is absorbs no row of $\boldsymbol{U}$. Other subsets are not in π.

Let $t(\boldsymbol{x}^*, \boldsymbol{U})$ be the set of numbers of those rows of $\boldsymbol{U}$, which absorb $\boldsymbol{x}^*$. For every block π_j of π, we define the Boolean function $\pi_j(\boldsymbol{x})$ having assumed that $\pi_j(\boldsymbol{x}^*) = 1$ for any $\boldsymbol{x}^* \in \{0,1\}^n$ if $t(\boldsymbol{x}^*, \boldsymbol{U}) = \pi_j$, and $\pi_j(\boldsymbol{x}^*) = 0$ otherwise.

Obviously, the cover π is unique for the matrix $\boldsymbol{U}$. The disjunction of all the Boolean functions assigned to the blocks of a cover is equal to 1, and the conjunction of any two Boolean functions assigned to different blocks of a cover is equal to 0, i.e. those functions are mutually orthogonal.

Let matrices $\boldsymbol{U}_1$ and $\boldsymbol{U}_2$ with the common set L of row numbers have covers π^1 and π^2 respectively. Let us form the set $\lambda = \{\pi^1_i \cap \pi^2_j \,/\, \pi^1_i \in \pi^1,\ \pi^2_j \in \pi^2,\ \pi^1_i(\boldsymbol{x}) \wedge \pi^2_j(\boldsymbol{x}) \neq 0\}$. For any element $\lambda_{ij} = \pi^1_i \cap \pi^2_j$ of λ, let $\lambda_{ij}(\boldsymbol{x}) = \pi^1_i(\boldsymbol{x}) \wedge \pi^2_j(\boldsymbol{x})$. We construct the cover π', having taken all the different elements of λ as elements of π'. For each block π'_s of π', let us define the Boolean function $\pi'_s(\boldsymbol{x})$ as the disjunction of all the Boolean functions assigned to those elements of λ, which are equal to π'_s. The cover π' is the *product of the covers* π^1 and π^2 ($\pi' = \pi^1 \times \pi^2$). The product operation gives a possibility to suggest a simple way to calculate the cover of a ternary matrix $\boldsymbol{U}$. This way consists of obtaining products of trivial covers of all the one-column matrices representing the columns of $\boldsymbol{U}$ (*Pottosin, 2006).*

Let us define an operation $\vee(\pi_i, \boldsymbol{V})$ over the rows of a binary matrix $\boldsymbol{V}$, the result of which is the vector $\boldsymbol{y}^*$ ($\boldsymbol{y}^* = \vee(\pi_i, \boldsymbol{V})$) obtained by component-wise disjunction of rows $\boldsymbol{V}$ whose numbers are in the block π_i. If $\pi_i = \varnothing$, all the components of $\boldsymbol{y}^*$ are equal to 0. It is shown in (*Pottosin, 2006) that* $\boldsymbol{f}(\boldsymbol{x}^*) = \boldsymbol{y}^* = \vee(\pi_i, \boldsymbol{V})$ if $\pi_i(\boldsymbol{x}^*) = 1$.

There is a convenient technique to construct the cover of a ternary matrix $\boldsymbol{U}$ when the number of arguments is not large. This technique uses the *cover map* that has the structure of the Karnaugh map. In any cell of a cover map of $\boldsymbol{U}$ corresponding to a vector $\boldsymbol{x}^*$, there is the set $t(\boldsymbol{x}^*, \boldsymbol{U})$, which is a block of the cover of $\boldsymbol{U}$.

Example 1. Figure 1 shows the cover map of the matrix

$$\boldsymbol{U} = \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 \end{matrix} \\ \begin{bmatrix} 0 & - & 0 & 1 \\ - & 1 & 0 & 1 \\ 1 & 0 & - & - \end{bmatrix} \end{array} \begin{matrix} \\ 1 \\ 2 \\ 3 \end{matrix}.$$

The cover is $\{\varnothing, \{1\}, \{2\}, \{3\}, \{1, 2\}\}$. Using this cover map, as well as using the Karnaugh map, the following DNFs of Boolean functions can be easily obtained: $\pi^1_1(\boldsymbol{x}) = \bar{x}_1\,\bar{x}_4 \vee \bar{x}_1 x_3 \vee x_2\,\bar{x}_4 \vee x_2 x_3$, $\pi^1_2(\boldsymbol{x}) = \bar{x}_1\,\bar{x}_2\,\bar{x}_3 x_4$, $\pi^1_3(\boldsymbol{x}) = x_1 x_2\,\bar{x}_3 x_4$, $\pi^1_4(\boldsymbol{x}) = x_1\,\bar{x}_2$, $\pi^1_5(\boldsymbol{x}) = \bar{x}_1 x_2\,\bar{x}_3\,x_4$.

x_4, x_3 (columns); x_2, x_1 (rows)

∅	∅	∅	1
∅	∅	∅	1,2
∅	∅	∅	2
3	3	3	3

Figure 1: An example of a cover map.

Let a pair of matrices, $\boldsymbol{U}$ and $\boldsymbol{V}$, give a system of completely specified Boolean functions $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})$, and let the matrix $\boldsymbol{U}_1$ be composed of the columns of $\boldsymbol{U}$, marked with the variables from the set $\boldsymbol{Z}_1$ and the matrix $\boldsymbol{U}_2$ from the columns marked with the variables from $\boldsymbol{Z}_2$. The covers of $\boldsymbol{U}_1$ and $\boldsymbol{U}_2$ are $\pi^1 = \{\pi^1_1, \pi^1_2, \ldots, \pi^1_r\}$ and $\pi^2 = \{\pi^2_1, \pi^2_2, \ldots, \pi^2_s\}$. Let us construct a table M. Assign the blocks $\pi^1_1, \pi^1_2, \ldots, \pi^1_r$ and the Boolean functions $\pi^1_1(\boldsymbol{z}_1), \pi^1_2(\boldsymbol{z}_1), \ldots, \pi^1_r(\boldsymbol{z}_1)$ to the columns of M, and $\pi^2_1, \pi^2_2, \ldots, \pi^2_s$ and $\pi^2_1(\boldsymbol{z}_2), \pi^2_2(\boldsymbol{z}_2), \ldots, \pi^2_s(\boldsymbol{z}_2)$ to the rows of M. At the intersection of the i-th column, $1 \le i \le r$ and the j-th row, $1 \le j \le s$, of M, we put the value $\boldsymbol{y}^* = \vee(\pi^1_i \cap \pi^2_j, \boldsymbol{V})$. The table M is called the *compact table*. It gives the system of Boolean functions $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})$ in the following way: the value of the vector Boolean function $\boldsymbol{f}(\boldsymbol{x}^*)$ is $\vee(\pi^1_i \cap \pi^2_j, \boldsymbol{V})$ at any set argument values $\boldsymbol{x}^*$, for which $\pi^1_i(\boldsymbol{z}_1) \wedge \pi^2_j(\boldsymbol{z}_2) = 1$. The form of the compact table is the same as that of the known decomposition chart (Perkowski, 1995). The difference is only that the rows and the columns of a decomposition chart correspond to sets of values of arguments, and the rows and the columns of a compact table to pair-wise disjoint families of such sets.

Having the compact table for a system of functions $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})$, it is easy to construct the desired systems $\boldsymbol{y} = \boldsymbol{\varphi}(\boldsymbol{w}, \boldsymbol{z}_2)$ and $\boldsymbol{w} = \boldsymbol{g}(\boldsymbol{z}_1)$. The columns of the compact table are encoded with binary codes; equal columns may have the same codes. The length of the code is equal to $\lceil r' \rceil$ where r' is the number of different columns of the table and $\lceil r' \rceil$ is the least integer, which is not less than $\log_2 r'$. So, the system of functions $\boldsymbol{w} = \boldsymbol{g}(\boldsymbol{z}_1)$ is defined. The value of the vector variable $\boldsymbol{w}$ at any set of values of the vector variable $\boldsymbol{z}_1$ turning the function $\pi^1_i(\boldsymbol{z}_1)$ into 1 is the code of the i-th column, $1 \le i \le r$. Naturally, there is no solution to this task at the given partition $\{Z_1, Z_2\}$ of the set X of arguments if the length of the code is not less than the length of $\boldsymbol{z}_1$. Otherwise, the compact table whose columns are assigned with the values of the variable $\boldsymbol{w}$

can be considered as a form of representation of the other desired system of functions $\boldsymbol{y} = \boldsymbol{\varphi}(\boldsymbol{w}, \boldsymbol{z}_2)$. The value of $\boldsymbol{y}$ at the value of $\boldsymbol{w}$ assigned to the i-th column, $1 \le i \le r$, and at any value of $\boldsymbol{z}_2$ turning $\pi^2_j(\boldsymbol{z}_2)$ into 1, $1 \le j \le s$, is the vector that is at the intersection of the i-th column and the j-th row.

Example 2. Let a system of completely specified functions $\boldsymbol{y} = \boldsymbol{f}(\boldsymbol{x})$ be given by the following pair of matrices:

$$\boldsymbol{U} = \begin{bmatrix} 0 & 0 & 0 & 1 & - \\ 0 & 1 & 0 & 0 & - \\ 0 & 1 & - & 0 & 1 \\ 0 & - & 0 & 0 & - \\ 0 & 0 & - & 0 & 1 \\ 1 & 1 & 0 & 1 & - \\ 1 & 1 & - & 1 & 1 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix}, \quad \boldsymbol{V} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix}.$$

(columns of $\boldsymbol{U}$: $x_1\ x_2\ x_3\ x_4\ x_5$; columns of $\boldsymbol{V}$: $y_1\ y_2$)

For the partition of the set of arguments into subsets $Z_1 = \{x_1, x_2, x_3\}$ and $Z_2 = \{x_4, x_5\}$, we have the following matrices:

$$\boldsymbol{U_1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & - \\ 0 & - & 0 \\ 0 & 0 & - \\ 1 & 1 & 0 \\ 1 & 1 & - \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix}, \quad \boldsymbol{U_2} = \begin{bmatrix} 1 & - \\ 0 & - \\ 0 & 1 \\ 0 & - \\ 0 & 1 \\ 1 & - \\ 1 & 1 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix}.$$

(columns of $\boldsymbol{U_1}$: $x_1\ x_2\ x_3$; columns of $\boldsymbol{U_2}$: $x_4\ x_5$)

To find the length of $\boldsymbol{w}$ in the superposition $\boldsymbol{y} = \boldsymbol{\varphi}(\boldsymbol{w}, \boldsymbol{z}_2)$, $\boldsymbol{w} = \boldsymbol{g}(\boldsymbol{z}_1)$ where $\boldsymbol{z}_1 = (x_1, x_2, x_3)$ and $\boldsymbol{z}_2 = (x_4, x_5)$, we construct the covers of the ternary matrices $\boldsymbol{U}_1$ and $\boldsymbol{U}_2$: $\pi^1 = \{\varnothing, \{3\}, \{5\}, \{7\}, \{6, 7\}, \{1, 4, 5\}, \{2, 3, 4\}\}$ and $\pi^2 = \{\{1, 6\}, \{2, 4\}, \{1, 6, 7\}, \{2, 3, 4, 5\}\}$. The corresponding compact table is represented in Table 1 that has seven different columns. Clearly, this task has no solution at the given subsets Z_1 and Z_2, because to encode the columns of the compact table with the values of $\boldsymbol{w}$, three variables are needed that is not less than the length of $\boldsymbol{z}_1$.The below is an example of Table. Note that the caption should be above the table.

Table 1: The compact table for the system of functions in Example 1.

	∅	3	5	7	6,7	1,4,5	2,3,4
1,6	00	00	00	00	01	10	00
2,4	00	00	00	00	01	01	11
1,6,7	00	00	00	01	01	10	00
2,3,4,5	00	10	01	00	00	11	11

4 SEARCH FOR A PARTITION OF THE SET OF ARGUMENTS

To search for an appropriate partition of the set of arguments that results in a solution of the task of decomposition of a system of completely specified Boolean functions, we use ternary matrix covers and compact tables induced by them.

It was said above that the cover of a ternary matrix $\boldsymbol{U}$ can be obtained using the product operation on the trivial covers of all the one-column matrices that are the columns of $\boldsymbol{U}$. The cover of any column of a ternary matrix consists of exactly two blocks – one of them contains numbers of one-element rows having 0s and "–" symbols, the other contains numbers of rows having 1s and "–" symbols. If a column consists only of 0s or only of 1s, then one of the blocks is empty. So, if a matrix $\boldsymbol{U}$ has n columns, then its cover π can be obtained as $\pi = \pi^1 \times \pi^2 \times \ldots \times \pi^n$ where π^i is the cover of the i-th column, $1 \le i \le n$.

Let a few free variables be to find that constitute the set Z_2 (then the set of bound variables would be $Z_1 = X \setminus Z_2$). To do this, we use the operation of dividing a ternary matrix cover by the cover of a column of the matrix.

Let us determine the operation to divide the cover π of a ternary matrix $\boldsymbol{U}$ by the cover π^i of its i-th column as $\pi / \pi^i = \pi^1 \times \pi^2 \times \ldots \times \pi^{i-1} \times \pi^{i+1} \times \ldots \times \pi^n$. This operation can be easily fulfilled using the *cover map*, which, as well as Karnaugh map, has the lines of symmetry related to the variables of the Boolean space represented by this map (Zakrevskij, 2007). To transform the cover map of a ternary matrix $\boldsymbol{U}$ into that of the matrix obtained from $\boldsymbol{U}$ by deleting the i-th column, one should superpose pair-wise the entries that are symmetric with regard to the lines relative to x_i, and put the unions of the superposed entries into the obtained entries. The obtained cover map would represent the desired cover.

Example 3. Figure 2 shows the cover map the ternary matrix $\boldsymbol{U}$ from Example 2. The cover of $\boldsymbol{U}$ is $\pi = \{\varnothing, \{1\}, \{3\}, \{4\}, \{5\}, \{6\}, \{7\}, \{2, 4\}, \{4, 5\}, \{6, 7\}, \{2, 3, 4\}\}$. The division of π by the cover of the column x_1 does not change π. It is seen from the cover map in Figure 3. Having transformed this map by the described way with regard to x_2, we obtain $\{\varnothing, \{7\}, \{1, 6\}, \{2, 4\}, \{3, 5\}, \{1, 6, 7\}, \{2, 3, 4, 5\}\}$ as a result of dividing π by the covers of the columns x_1 and x_2 (see Figure 4).

Continuation of Example 3. Let us attempt to obtain a decomposition of the given system of function in the form of $\boldsymbol{y} = \boldsymbol{\varphi}(\boldsymbol{w}, \boldsymbol{z}_2)$, $\boldsymbol{w} = \boldsymbol{g}(\boldsymbol{z}_1)$ where $\boldsymbol{z}_1 = (x_3, x_4, x_5)$ and $\boldsymbol{z}_2 = (x_1, x_2)$. The corresponding compact table is Table 2 that has five different columns, and this task has no solution at that partition.

x_5 x_4 x_3

4	∅	∅	1	1	∅	5	4,5
2,4	∅	∅	∅	∅	∅	3	2,3,4
∅	∅	∅	6	6,7	7	∅	∅
∅	∅	∅	∅	∅	∅	∅	∅

x_2 x_1

Figure 2. The cover map of matrix ***U*** from Example 2

x_5 x_4 x_3

4	∅	∅	1	1	∅	5	4,5
2,4	∅	∅	6	6,7	7	3	2,3,4

x_2

Figure 3. The cover map obtained by dividing π by the cover of the column x_1

x_5 x_4 x_3

2,4	∅	∅	1,6	1,6,7	7	3,5	2,3,4,5

Figure 4. The cover map obtained by dividing π by the covers of the column x_1 and x_2

Table 2: The compact table for the partition in Example 3.

	∅	7	1,6	2,4	3,5	1,6,7	2,3,4,5
∅	00	00	00	00	00	00	00
6,7	00	01	01	00	00	01	00
1,4,5	00	00	10	01	01	10	01
2,3,4	00	00	00	11	10	00	11

The method suggested for the search for an appropriate partition consists in fulfilling the lexicographical enumeration and testing by the above way every variant of the set Z_2 if it would provide a solution of the task.

Example 4. Let the system of completely specified Boolean functions from Example 2 be given. Fulfilling the lexicographical enumeration of variants of Z_2 that consist of two variables, we find that the first appropriate variant is $Z_2 = \{x_2, x_4\}$, $Z_1 = \{x_1, x_3, x_5\}$. For this variant with the cover map in Figure 2, we obtain the cover map shown in Figure 5 and then we obtain Figure 6 from Figure 5.

x_5 x_4 x_3

2,4	∅	∅	1	1	∅	3,5	2,3,4,5
∅	∅	∅	6	6,7	7	∅	∅

x_2

Figure 5. The cover map obtained by dividing π by the cover of the column x_2

The compact table for the covers $\pi^1 = \{\varnothing, \{6\}, \{7\}, \{3, 5\}, \{6, 7\}, \{1, 2, 4\}, \{1, 2, 3, 4, 5\}\}$ and $\pi^2 = \{\{1\}, \{4, 5\}, \{6, 7\}, \{2, 3, 4\}\}$ is represented by Table 3 that have four different columns. To encode these columns, two variables are sufficient. The codes of the columns are shown at the bottom of Table 3.

x_5 x_3

1,2,4	∅	3,5	1,2,3,4,5
6	∅	7	6,7

x_2

Figure 6. The cover map obtained by dividing π by the covers of the column x_2 and x_4

Table 3: The compact table for the partition from Example 4.

	∅	6	7	3,5	6,7	1,2,4	1,2,3,4,5
1	00	00	00	00	00	10	10
4,5	00	00	00	01	00	01	01
6,7	00	01	01	00	01	00	00
2,3,4	00	00	00	10	00	11	11
	00	01	01	10	01	11	11

To construct the systems of functions $\boldsymbol{y} = \boldsymbol{\varphi}(\boldsymbol{w}, \boldsymbol{z}_2)$ and $\boldsymbol{w} = \boldsymbol{g}(\boldsymbol{z}_1)$ that are the solution of the task, the functions connected with the blocks of the covers obtained must be constructed. The DNFs of the functions connected with the blocks of π^1 can be obtained from the cover map in Figure 6: $\pi^1_1(\boldsymbol{z}_1) = x_3 \bar{x}_5$, $\pi^1_2(\boldsymbol{z}_1) = x_1 \bar{x}_3 \bar{x}_5$, $\pi^1_3(\boldsymbol{z}_1) = x_1 x_3 x_5$, $\pi^1_4(\boldsymbol{z}_1) = \bar{x}_1 x_3 x_5$, $\pi^1_5(\boldsymbol{z}_1) = x_1 \bar{x}_3 x_5$, $\pi^1_6(\boldsymbol{z}_1) = \bar{x}_1 \bar{x}_3 \bar{x}_5$, $\pi^1_7(\boldsymbol{z}_1) = \bar{x}_1 \bar{x}_3 x_5$. Similarly, the DNFs $\pi^2_1(\boldsymbol{z}_2) = \bar{x}_2 x_4$, $\pi^2_2(\boldsymbol{z}_2) = \bar{x}_2 \bar{x}_4$, $\pi^2_3(\boldsymbol{z}_2) = x_2 x_4$, $\pi^2_4(\boldsymbol{z}_2) = x_2 \bar{x}_4$. are obtained. As a result of simple minimization we obtain the following matrices representing the desired superposition $\boldsymbol{y} = \boldsymbol{\varphi}(\boldsymbol{w}, \boldsymbol{z}_2)$, $\boldsymbol{w} = \boldsymbol{g}(\boldsymbol{z}_1)$:

$$\begin{array}{cccc} w_1 & w_2 & x_2 & x_4 \end{array} \quad \begin{array}{cc} y_1 & y_2 \end{array} \qquad \begin{array}{ccc} x_1 & x_3 & x_5 \end{array} \quad \begin{array}{cc} w_1 & w_2 \end{array}$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & - & 1 & 0 \\ 1 & - & 0 & 0 \\ 1 & 1 & - & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}; \quad \begin{bmatrix} 0 & 0 & - \\ - & 0 & 0 \\ 1 & - & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}.$$

If we are intended to minimize the length of the vector ***w***, we should continue the search. For this example, the length of ***w*** cannot be reduced, but there is another variant, $Z_2 = \{x_3, x_5\}$, $Z_1 = \{x_1, x_2, x_4\}$, for which the length of ***w*** is equal to 2 as well.

5 CONCLUSION

The method suggested for searching for a partition of the set of arguments for decomposition of a system of Boolean functions does not exclude the complete enumeration of different variants of the partition, but using ternary matrices covers and the representation of a system of Boolean functions in the form of compact table allows rather simple testing the partitions to be appropriate for decomposition.

Reference

Bibilo P.N. (2009). *Decomposition of Boolean Functions Based on Solving Logical Equations*. Minsk: Byelaruskaya Navuka (in Russian).

Jóźwiak L. and A. Chojnacki (2000). “An Effective and Efficient Method for Functional Decomposition of Boolean Functions Based on Information Relationship Measures”. In Design and Diagnostics of Electronic Circuits and Systems. Proceedings of 3rd DDECS Workshop. Bratislava: Institute of Informatics, Slovak Academy of Sciences, pp. 242-249.

Perkowski M.A. and S. Grygiel (1995). *A Survey of Literature on Functional Decomposition. Version IV (Technical report)*. *Portland: Portland State University, Department of Electrical Engineering.*

Pottosin Yu.V. and E.A. Shestakov (2006). Tabular Methods for Decomposition of Systems of Completely Specified Boolean Functions. Minsk: Byelorusskaya Nauka (in Russian).

*Zakrevskij, A.D. (*2007). *□Decomposition of Partial Boolean Functions: Testing for Decomposability According to a Given Partition□. In* Informatika, No. 1(13), pp. 16-21 (in Russian).

Zakrevskij, A., Yu. Pottosin and L. Cheremisinova (2009). Optimization in Boolean Space. Tallinn: TUT Press.