10.- Bayesian Games (II)

48. **[Bayesian Hawk and Dove]** Two players compete for a resource of value V (food, land, etc.) Each one can behave aggressively (Hawk) or peacefully (Dove).

    If two Doves meet, the resource is split equally among them (each receives $V/2$). If a Dove meets a Hawk, the latter takes all (V) and the Dove flees empty-handed (0). If two Hawks meet, there is a fight, and each gets the resource with probability 1/2. As a simplification, assume each obtains $V/2$ (interpreted as expectation). However, the fight produces a damage of value -2 for each Hawk.

    The value of the resource can be either $V = 2$ or $V = 6$. That is, we consider the two payoff matrices:

    Matrix 1 ($V = 2$).

	H	D
H	-1,-1	2,0
D	0,2	1,1

    Matrix 2 ($V = 6$).

	H	D
H	1,1	6,0
D	0,6	3,3

    The true value of V is determined at random: $V = 2$ happens with probability p, and $V = 6$ with probability $1 - p$. Player 1 knows whether $V = 2$ or $V = 6$, but player 2 does not (he knows only the probabilities).

    (a) Consider this situation as a Bayesian game. Describe the types $t_i \in T_i$, the actions $a_i \in A_i$, the beliefs p_i, and the payoff functions π_i. Describe also the strategies $s_i \in S_i$.

    (b) Write down the Extensive Form of this game as a game of complete but imperfect information.

    (c) Find the Bayes-Nash equilibria of this game (in pure strategies).

49. **[Public Good]** Consider the problem of public good provision described in class with two players, each of them with two possible actions—contribute (C) and not contribute (N)—and payoffs as in the following payoff matrix.

$1 \setminus 2$	C	N
C	$1 - c_1, 1 - c_2$	$1 - c_1, 1$
N	$1, 1 - c_2$	$0, 0$

    Suppose player 1 may have cost $c_1 \in \{0.75, 1.5\}$ and player 2 may have cost $c_2 \in \{0, 0.75\}$. Each player observes her own cost, but not the opponent's cost. Let $0 < p < 1$ be the probability that player $i = 1, 2$ attaches to player $j \neq i$ having cost $c_j = 0.75$; i. e. type 0.75 occurs with probability p for both players and costs are drawn independently.

    (a) Explain and sketch Harsanyi's transformation. What would be Nature's "strategy" in this case?

    (b) Define Bayesian equilibrium for this game.

    (c) Calculate all pure-strategy Bayesian equilibria and explain each of them intuitively.

50. **[Continuum of Types]** Consider the game with the following payoff matrix. The value of x_i is observable to player $i \in \{1, 2\}$ only. Suppose that x_1 and x_2 are independently and uniformly distributed in the interval $[-1, 1]$.

$1 \setminus 2$	A	B
A	$0, 0$	$1, x_2$
B	$x_1, 1$	$0, 0$

    (a) Show that there is no Bayesian equilibrium where one of the players always plays B; i. e. where, for some $i \in \{1, 2\}$, $s_i(x_i) = B$ for all $x_i \in [-1, 1]$.

(b) Show that the following pair of strategies is a Bayesian equilibrium, (s_1, s_2) with $s_1(x_1) = A$ and

$$s_2(x_2) = \begin{cases} \text{A} & \text{if } x_2 \leq 0 \\ \text{B} & \text{otherwise} \end{cases}$$

(c) Find at least another Bayesian equilibrium.

Homework:

51. Consider the game with the following payoff matrix, where the value of x is only observable to player 2.

		Player 2 A	Player 2 B
Player 1	A	$0, 0$	$1, x$
	B	$1, 1$	$0, 0$

Calculate all pure-strategy Bayesian equilibria in each of the following two cases.

(a) Player 1 believes that $x = 1$ with probability $0 < p < 1$ and $x = -1$ with probability $(1 - p)$.

(b) Player 1 believes that x is uniformly distributed in the interval $[-1, 1]$.