Probabilistic, weight-dependent STDP leads to rate-dependent synaptic fixed points

DOMINIC STANDAGE AND THOMAS TRAPPENBERG

Faculty of Computer Science, Dalhousie University

`standage | tt @cs.dal.ca`

1 Introduction

Change in synaptic efficacy is believed to underlie learning and memory, and has long been established in the forms of long term potentiation (LTP) and long term depression (LTD) [2]. Experimental and theoretical work on plasticity has addressed the dependence of plasticity on presynaptic firing rates [1, 3] and the timing [7] and interaction [4, 6] of pre- and post-synaptic spikes. Computational studies have further explored the consequences of plasticity on the distribution of synaptic strengths [10, 6] but this work has not clearly addressed correlated pre- and post-synaptic activity. An important issue for modelling plasticity is limiting synaptic strength following correlated activity, as correlations among input spikes can drive synapses without bound. Modelling studies have used hard [8] and soft [5] caps on synaptic strength to address this issue.

We recently proposed a new interpretation of Bi and Poo's [7] spike-time-dependent plasticity (STDP) data, deriving a weight- and spike-time-dependent plasticity rule that leads to fixed points of synaptic weights following correlated pre- and post-synaptic spike trains [9]. Under our rule, fixed points are achieved by a balance of LTP and LTD, where LTD increases faster than LTP and therefor dominates strong synapses. These fixed points are dependent on the rate and distribution of pre-synaptic spikes, post-synaptic spike latency, and the probability of synaptic transmission. Here, we build on this work by adding a weight-dependent probabilistic term to our plasticity rule. Without this term, fixed points are several orders of magnitude larger than experimental values, and our rule leads to runaway weights when LTP is isolated from LTD, as is typically the case in STDP protocols. Accordingly, input associations quickly lead to weights capable of driving the post-synaptic neuron on their own. We show that our new, probabilistic rule retains the features of our earlier rule, but lowers fixed points to biologically realistic values. We further demonstrate that associations between pre-synaptic events driving a model neuron are captured by this rule, so that several synapses are on average required to drive a post-synaptic neuron following associative learning, commensurate with biological findings.

2 Methods and Results

We consider the weight- and spike-time-dependence of synaptic change, assuming independence between these factors for simplicity. Our learning rule is

$$\Delta w_{\{p,d\}} = a_{\{p,d\}} w^{b_{\{p,d\}}} e^{-c_{\{p,d\}} \Delta t} \gamma(w), \tag{1}$$

where Δw is the change in synaptic strength (weight), $\Delta t = t_{post} - t_{pre}$ is the temporal difference between post-synaptic (t_{post}) and pre-synaptic (t_{pre}) spikes, γ is a weight-dependent

Bernoulli random variable, and constants $a_{\{p,d\}} = \{431, -59\}/60$, $b_{\{p,d\}} = \{0.4, 0.1\}$, and $c_{\{p,d\}} = \{0.039, 0.043\}$ were derived by fitting Bi and Poo's weight- and spike-time-dependent data under the assumption that each of their 60 spike pairings [7] contributed equally to the measured synaptic change.

The maximally correlated case between a pre-synaptic event and postsynaptic spike results in an ongoing potentiation of synaptic weights, but every pre-synaptic event can also trigger depression in conjunction with a previous post-synaptic spike. For Poisson-distributed correlated pre- and post-synaptic spikes, an equilibrium value w^* is given by

$$e^{w^*/(b_d-b_p)\tau_\gamma} w^* = (-\frac{a_p}{a_d}\frac{1}{r} e^{-c_p \Delta t}\,(r + c_d))^{\frac{1}{b_d-b_p}}, \tag{2}$$

where $\tau_\gamma = 500\text{ms}$ was chosen because LTP was not reliably produced when initial synaptic strengths were greater than 500pA in [7].

Figure 1A shows the effects of LTP under Equation 1 in the absence of LTD, with and without the probabilistic term γ. Without γ, Equation 1 leads to runaway LTP because large weights are potentiated more than small weights (despite their *percentage* potentiation being smaller). With the inclusion of γ, LTP leads to weight values on the order of those found in [7]. Figure 1B shows fixed points as a function of firing rate for correlated and uncorrelated pre- and post-synaptic spike trains. Solid and dashed curves show fixed points with and without the inclusion of γ respectively. These equilibrium values depend on rate, correlation, and spike latency.

Figure 1C shows the effect of correlated input activity on synaptic weights under Equation 1. Fifty correlated inputs representing a specific pre-synaptic event drive a leaky integrate-and-fire (LIF) node, where synaptic weights associated with this event are initialised to 100pA. After a few instances of the event, weights have increased enough that five synapses are able to drive the node. Small weights (10pA) that are active during the LTP time window due to a second event (onset at $t = 10^4$ms) are also potentiated, and soon five of these synapses are able to drive the neuron in the absence of the first event. Weight values quickly plateau, as seen in the figure.

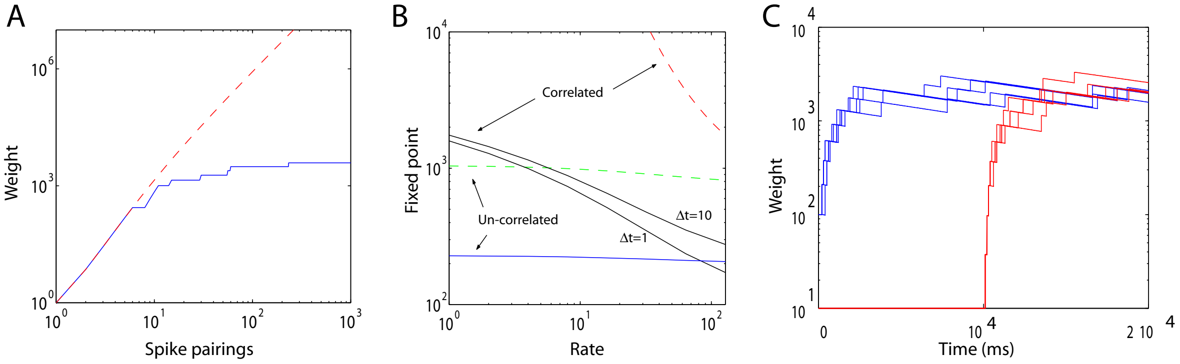

Figure 1: (A) In the absence of LTD, Equation 1 leads to runaway weights (dashed line) when $\gamma = 1$. When γ is w-dependent, LTP leads to stable weights (solid). (B) Equation 1 leads to synaptic fixed points that depend on the rate and correlation of spike trains. Curves depict these dependencies with (solid) and without (dashed) the weight-dependent probabilistic term γ. (C) 50 correlated inputs (initial weight 100pA, 5 shown) drive an LIF node leading to potentiation under Equation 1. Five synapses (left) are soon able to drive the post-synaptic node. Five weak synapses (10pA) then piggyback this pre-synaptic event and are also potentiated (right).

3 Conclusions

In their classic paper [7] Bi and Poo state that they limit their experiments to synapses with initial strengths less than 500pA because they were unable to reliably elicit LTP for larger synapses. Because their weight-dependent data show significant LTP for several synapses with initial strengths of around 1000pA, we hypothesise that LTP may be probabilistic, depending on initial strength. This hypothesis could be tested under established STDP protocols [7, 4]. By adding a weight-dependent, probabilistic term to our weight- and spike-time-dependent plasticity rule in [9] we provide an STDP rule that leads to rate-dependent synaptic fixed points on the order of experimental values. Our rule retains the features shown in [9], leads to stable LTP without recourse to a cap on synaptic strength, and captures associations among input spikes driving a model neuron.

References

[1] E. L. Bienenstock, L. N. Cooper, and P. W. Munro. Theory for the development of neuron selectivity: orientation specificity and binocular interaction in visual cortex. *Journal of Neuroscience*, 2(1):32–48, 1982.

[2] T. V. P. Bliss and T. J. Lomo. Long-lasting potentiation of synaptic transmission in the dentate area of the anaesthetized rabbit following stimulation of the perforant path. *Journal of Physiology*, 232:331–356, 1973.

[3] S. M. Dudek and M. F. Bear. Homosynaptic long-term depression in area ca1 of hippocampus and effects of n-methyl-d-aspartate receptor blockade. *Proceedings of the National Academy of Sciences of the United States of America*, 89:4363–4367, 1992.

[4] R. C. Froemke and Y. Dan. Spike-timing-dependent synaptic modification induced by natural spike trains. *Nature*, 416(6879):433–438, 2002.

[5] R. Gutig, R. Aharonov, S. Rotter, and H. Sompolinsky. Learning input correlations through nonlinear temporally asymmetric hebbian plasticity. *Journal of Neuroscience*, 23:3697–3714, May 2003.

[6] E. M. Izhikevich and N. S. Desai. Relating stdp to bcm. *Neural Computation*, 15(7):1511–1523, 2003.

[7] G. qiang Bi and M. ming Poo. Synaptic modifications in cultured hippocampal neurons: Dependence on spike timing, synaptic strength, and postsynaptic cell type. *Journal of Neuroscience*, 18:10464–10472, 1998.

[8] S. Song, K. D. Miller, and L. F. Abbott. Competitive hebbian learning through spike-timing-dependent synaptic plasticity. *Nature Neuroscience*, 3(9):919–926, 2000.

[9] T. Trappenberg, D. Standage, and S. Jalil. Associative learning by a physiologically derived stdp rule leads to synaptic fixed points. Submitted to Neural Computation, January 2006.

[10] M. C. W. van Rossum, G. Q. Bi, and G. G. Turrigiano. Stable hebbian learning from spike timing-dependent plasticity. *Journal of Neuroscience*, 20(23):8812–8821, 2000.