Reconsidering the Jeep Problem
- Or How to Transport a Birthday Present to Salosauna -

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Abstract

The simple problem of how far a jeep can travel with a given amount of gasoline if intermediate gasoline dumps may be used is a nice example for problems which seem to have obvious recursive solution algorithms but which may become quite difficult if the problem specification is slightly changed. The classical version allows that arbitrarily small parts of the given amount of gasoline may be filled in the jeep's tank. Wood has restricted the problem to a discrete problem by requiring that the tank can be refilled only when it is empty and that it must be refilled completely, and that the gasoline is available only in cans of the size of the tank. In an earlier note we had shown, by using a new strategy, that the seemingly adequate algorithm given by Wood in analogy to the optimal solution to the classical version is not optimal. The new strategy however is also not optimal. In this note we discuss variants of the new strategy and try to get a better understanding of the influences of (small) changes in the problem specification to the solution algorithms.

The Problem and Earlier Results

The classical jeep problem (see [Fin], [Phi], [Gal]) is to compute how far a jeep may go when starting at a dump with n cans of gasoline, provided that it needs 1 canful to drive 1 unit distance and it is allowed to carry 2 canfuls of gasoline (including the contents of its tank). The optimal solution algorithm (see [Gal]) assumes that arbitrary fractions of a canful may be put into the tank.

Since this seems to be a bit unrealistic, a discrete variant was considered by D. Wood in [Woo]: It is only allowed to refill the tank, when it is empty and then 1 canful has to be filled in (thus 1 can can be transported). The following solution was proposed in [Woo]: With one canful transport $n-1$ cans to the next dump and go on recursively. This gives the distance function $f_w(n) = 1+1+\frac{1}{3}+\frac{1}{5}+\ldots+1/(2n-3)$. This seemed to be the natural adaptation of the classical optimal solution.

However in [Bra] we showed that $f_w(n)$ is not optimal for $n \geq 4$ by using a new strategy: With one canful transport only 1 can as far as possible. This means that, for even n, each second can is transported to a dump $\frac{1}{2}$ unit distance away - apart from the last one, which is moved to an auxiliary dump $\frac{3}{4}$ units away, and which is fetched in the next round with the last $\frac{1}{2}$ canful. This strategy should only be applied to even can numbers since if $n = 2k+1$ then only k cans are used for the transportation of k cans and the last one is wasted.

Let $[x; n]$ denote the situation, that at position x there is a dump containing n cans. We then can describe the procedure by

$$[0; 2\cdot(2k-1)] \rightarrow [\frac{1}{2}; 2k-2]\&[\frac{3}{4}; 1] \rightarrow [1; k] \qquad (1)$$

which means that the jeep starts at the dump at position 0 which contains $2\cdot(2k-1)$ cans. It then (in a first round) transports in each of $2k-2$ round trips (each using up one tankful) one can to a dump at position $\frac{1}{2}$, and in the final trip one can is moved to position $\frac{3}{4}$, and the jeep stops at position $\frac{1}{2}$ with an empty tank. In a second round $k-1$ cans are moved to a dump at position 1 in $k-1$ round trips and one trip from position $\frac{1}{2}$ to position 1 which leaves $\frac{1}{2}$ canful in the tank such that the jeep may bring the can from position $\frac{3}{4}$ to position 1. This strategy gives better results than Wood's strategy for $n = 6$ and all $n \geq 12$. The distance function for this procedure is