Comparison and Regulation of Neuronal Synchronization for Various STDP Rules

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Abstract

We discuss effects of various experimentally supported STDP learning rules on frequency synchronization of two unidirectional coupled neurons systematically. First, we show that synchronization windows for all STDP rules can't be enhanced compared to constant connection under the same model. Then, we explore the influence of learning parameters on synchronization window and find optimal parameters that lead to the widest window. Our findings indicate that synchronization strongly depends on the specific shape and the parameters of the STDP update rules. Thus, we give some explanations by analyzing the synchronization mechanisms for various STDP rules finally.

Keywords: spike-timing dependent plasticity (STDP); synchronization window; learning parameters; learning rules; synaptic conductance; synchronization mechanisms

Introduction

Synchronous activity is a basic characteristic in the brain. It exists in many regions of the brain, such as CA1 of the hippocampus (Fell, Klaver et al. 2001), visual cortex (Womelsdorf, Fries et al. 2006) and cortical areas correlating with conscious perception (Melloni, Molina et al. 2007). It is known that synchronization is very important for information processing, such as predicting sensory input(Womelsdorf and Fries 2006), and information codes(Biederlack, Castelo-Branco et al. 2006). Moreover, synchronous activity plays a crucial role in epileptic activity (Chavez, Le Van Quyen et al. 2003; Queiroz and Mello 2007), modulation of neurons about attention (Liang, Bressler et al. 2003), memory and learning (Wagner 2001; Axmacher, Mormann et al. 2006) , and cognitive functions (Varela, Lachaux et al. 2001).

Since the discovery of LTP (long term potentiation) and LTD (long term depression) (Bliss and Lomo 1973; Linden and Connor 1995; Nicoll and Malenka 1995), it has been debated how synaptic modifications are correlated to neuron activities. Spike timing-dependent plasticity (STDP) is a form of synaptic modification discovered relatively recently, which depends on the relative timing of pre- and post-synaptic action potentials at a millisecond time scale (Gerstner and van Hemmen 1994; Gerstner, Kempter et al. 1996). Many experiments have proved the existence of STDP, such as in neocortical slices (Markram, Lübke et al. 1997), hippocampus slice (Debanne, Gahwiler et al. 1998), hippocampal cell cultures (Bi and Poo 1998), and tadpole rectum in vivo (Zhang, Tao et al. 1998). In addition, STDP provides powerful mechanisms for models of temporal pattern recognition (Gerstner, Ritz et al. 1993), temporal sequence learning (Minai and Levy 1993; Abbott and Blum 2003), a continuous-time associative memory (Watanabe, Watanabe et al. 2004), coincidence detection (Gerstner, Kempter et al. 1996; Gerstner, Kempter et al. 1997), navigation (Blum and Abbott 1999; Mehta, Quirk et al. 2000) and direction selectivity (Mehta and Wilson 2000).

The interaction among neurons relies much on synaptic modification in which STDP is the only one that greatly expands the capability of Hebbian learning to address temporally sensitive computational tasks. STDP in synchronization has attracted wide interests. For example, the result of learning-induced synchronization of a neural network at various developing stages using STDP rule is consistent with recent experimental observations (Chao and Chen 2005). Furthermore, the comparison of synchronization between discontinuous anti-STDP(dc-aSTDP, see section2) and constant connection has been investigated (Zhigulin, Rabinovich et al. 2003). Following it, the continuous STDP(c-STDP, see section 2) has also been studied (Nowotny, Zhigulin et al. 2003) by the same authors. They suggest that a functional role of STDP might be enhancing synchronization. Motivated by their work, we systemically discuss the roles of four types of STDP rules(c-STDP, dc-STDP, dc-aSTDP and in-STDP, see section2) in frequency synchronization in the present paper, employing the same model (Nowotny, Zhigulin et al. 2003) with only values of some parameters different, such as A_{plus}, t_{syn}, Vs_{lope}, g_{max} (see section 3).

We find, however, not all STDP rules facilitate synchronization. It encourages us to trace the reason. We then consider if the learning curve, which characterize the

STDP rule, have certain effects on synchronization. Results indicate that synchronization strongly depends on the specific shape and the parameters of the STDP rule. However, the optimal synchronization ranges for dc-STDP and in-STDP, got from regulating learning parameters, are not wider than those for the corresponding strongest constant connection respectively. As a result, when we seek the reason, we discover that the synchronization mechanisms of above four STDP rules can be classified into two categories: (i) c-STDP and dc-aSTDP rules; (ii) dc-STDP and in-STDP rules. The synchronization mechanisms of the two categories are different. For c-STDP and dc-aSTDP rules, two neurons' synchronization either relies on the balancing out potentiation and depression during one cycle consistent with the perspective of Nowotny et al, or relies on the maximal synaptic conductance. However, for dc-STDP and in-STDP rules, the synchronization windows are completely provided by the respective maximal synaptic conductance. As regards this finding, we offer an intuitive explanation finally.

Models and method

We consider two HH neurons with unidirectional activity-dependent excitatory or inhibitory synaptic coupling. Although such a configuration is too simple to find applications in brain information processing, it serves as a staring point for many model researches. The neurons are modeled with standard Na, K, and “leak” currents (Traub and Miles 1991),

$$C\frac{dV_i(t)}{dt} = -g_{Na}\cdot m_i(t)^3\cdot h_i(t)\cdot(V_i(t)-E_{Na}) - g_K\cdot n_i(t)^4\cdot(V_i(t)-E_K) - g_L\cdot(V_i(t)-E_L) - I_{syn}(t) + I_{stim},$$

where $i=1,2$.

Each of the activation and inactivation variables $y_i(t)=\{n_i(t), m_i(t), h_i(t)\}, i=1,2$ satisfies first-order kinetics,

$$\frac{dy_i(t)}{dt} = \alpha_y[V_i(t)][1\text{-}y_i(t)]\text{-}\beta_y[V_i(t)]y_i(t), i=1,2,$$

The parameters in these equations are given in(Nowotny, Zhigulin et al. 2003),

$$\begin{aligned}
\alpha_n &= 0.032(-50-V)/(\exp((-50-V)/5)-1)\\
\beta_n &= 0.5\exp((-55-V)40)\\
\alpha_m &= 0.32(-52-V)/(\exp((-52-V)/4)-1)\\
\beta_m &= 0.28(25+V)/(\exp((25+V)/5)-1)\\
\alpha_h &= 0.128\exp((-48-V)/18)\\
\beta_h &= 4/(\exp((-25-V)/5)+1)
\end{aligned}.$$

I_{stim} is a constant input current forcing each neuron to spike with a constant, I_{stim} -dependent period, labeled as T_1 and T_2. The postsynaptic neuron would show another firing period T_2^1, when it is driven by the synaptic current, which is dependent on the postsynaptic potential $V_2(t)$, the reversal potential V_{rev}, the activation variable $S(t)$ and its maximal conductance $g(t)$,

$$I_{syn}(t) = g(t)S(t)(V_2(t)\text{-}V_{rev}),$$

where

$$\frac{dS(t)}{dt} = \frac{S_\infty(V_1(t)) - S(t)}{t_{syn}\cdot(1-S_\infty(V_1(t)))}, \quad S_\infty(V) = \begin{cases}\tanh((V-V_{th})/V_{slope}), & for\ \ V > V_{th}\\ 0 & ,otherwise\end{cases}.$$

The time-dependent synaptic coupling strength $g(t)$ nS is

$$g(t) = \frac{g_{\max}}{2}(\tanh(\frac{g_{raw}-g_{mid}}{g_{slope}})+1).$$

Therefore *g(t)* always have values between 0 *nS* and g_{max}. The bound imposed on *g(t)* is artificially set to avoid unrealistically high synaptic conductance and negative conductance. In order to obtain biologically plausible synaptic conductance, several methods have been employed to limit the synaptic strength in the literature, such as a negative total integral (Kempter, Gerstner et al. 2001), artificial bounds (Song and Abbott 2001), and self-limitation (Nowotny, Zhigulin et al. 2003). Unless otherwise stated, we employ the self-limitation method which is characterized by a function 'tanh' in our simulation.

g_{raw} is modified by STDP rules that are introduced in the next paragraph. The initial value of g_{raw} is 20 *nS*. The parameters of the model are

$$C = 30\mu F; g_L = 1\mu S; E_L = -64mV; g_{Na} = 360\mu S; E_{Na} = 50mV; g_K = 70\mu S; E_K = -95mV;$$
$$V_{th} = -20mV; t_{syn} = 25ms; V_{slope} = 15mV; g_{\max} = 25nS; g_{mid} = \frac{1}{2} g_{\max}; g_{slope} = g_{mid}; V_{rev} = 20mV.$$

The time-dependent synaptic coupling strength *g(t)* is determined by the spike-timing of pre- and postsynaptic spikes. We consider four types of activity-dependent couplings that have been found in experiments: (1) an excitatory synapse with continuous STDP (c-STDP). There are two forms of c-STDP from two different experiments. One (Fig.1A) is from the recording of the neocortex-layer 5 Xenopus tectum hippocampus (Roberts and Bell 2002; Nowotny, Zhigulin et al. 2003), and the other is from the neocortex-layer 4 spiny stellates (Abbott and Nelson 2000). The latter form will not be considered here, because it introduces persistent decrease to synaptic strength that would result in none synchronization if two neurons have different inherent periods. (2) an excitatory synapse with discontinuous STDP (dc-STDP, Fig.1B) (Markram, Lübke et al. 1997; Abbott and Nelson 2000); (3) an excitatory synapse with discontinuous anti-STDP (Bell, Han et al. 1997; Abbott and Nelson 2000) (dc-aSTDP, Fig.1C); (4) an inhibitory synapse with STDP (in-STDP, Fig.1D) (Bi and Poo 1998; Debanne, Gahwiler et al. 1998; Abbott and Nelson 2000).

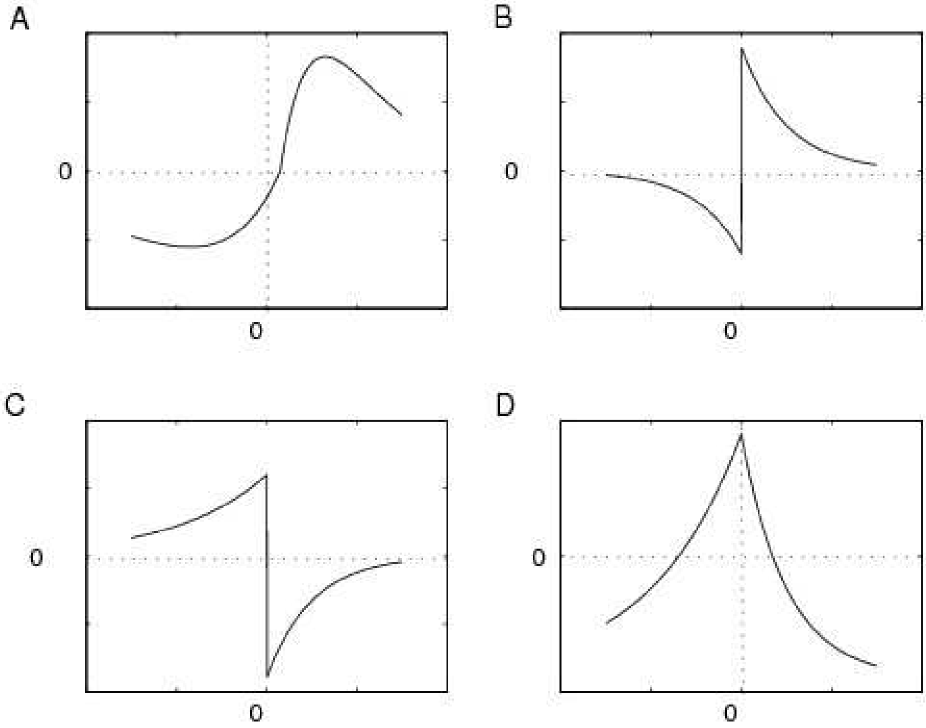

Figure 1: Different types of STDP curves are presented. A (c-STDP) B: discontinuous STDP(dc-STDP); C: discontinuous anti-STDP(dc-aSTDP); D: inhibitory STDP(in-STDP).

$\triangle g_{raw}$ is a function of $\triangle t = t_{postspike} - t_{prespike}$, time difference between the times of postsynaptic and pre-synaptic spikes. The learning rules corresponding to Fig.1 (A, B, C, D) are provided as follows c-STDP:

$$\Delta g_{raw}=\begin{cases} A_{plus}\dfrac{\Delta t-\tau_0}{t_{plus}}\cdot e^{\frac{-\Delta t}{t_{plus}}},\Delta t>\tau_0 \\ A_{sub}\dfrac{\Delta t-\tau_0}{t_{sub}}\cdot e^{\frac{\Delta t}{t_{sub}}},\Delta t\le\tau_0 \end{cases},$$

dc-STDP:

$$\Delta g_{raw}=\begin{cases} A_{plus}\cdot e^{\frac{-\Delta t}{t_{plus}}},\Delta t>0 \\ -A_{sub}\cdot e^{\frac{\Delta t}{t_{sub}}},\Delta t\le 0 \end{cases}$$

dc-aSTDP:

$$\Delta g_{raw}=\begin{cases} -A_{plus}\cdot e^{\frac{-\Delta t}{t_{plus}}},\Delta t>0 \\ A_{sub}\cdot e^{\frac{\Delta t}{t_{sub}}},\Delta t\le 0 \end{cases},$$

in-STDP:

$$\Delta g_{raw}=\begin{cases} A_{plus}\cdot(e^{\frac{-\Delta t}{t_{plus}}}-0.5),\Delta t>0 \\ A_{sub}\cdot(e^{\frac{\Delta t}{t_{sub}}}-0.5),\Delta t\le 0 \end{cases},A_{plus}=A_{sub}$$

Synchronization of pre- and post-synaptic neurons occur when $|T_1\text{-}<T_2^1>|$ is limited in an acceptable range. We set the criteria of synchronization as $|T_1\text{-}<T_2^1>|<1.5ms$. Although there is some arbitrariness in setting the criteria of synchronization, there is no qualitative change in our results if the criteria changes in two folds. Each simulation runs 20*s*, average is taken in the final 4000*ms*. We have observed that simulations from different initial values of V_2, S could result in different outputs, i.e. the post-synaptic neuron sometimes synchronizes with the pre-synaptic one, sometimes keep its original period, or sometimes fires with an oscillating period (see Fig.2C). We therefore carry out 40 times of simulations, from randomly selected initial values, for every T_2. The standard deviation of $|T_1 - \langle T_2^1 \rangle|$, indicating how precisely the neurons are synchronized, represents the quality of synchronization. Range of T_2, in which post-synaptic neuron is successfully entrained by the pre-synaptic neuron, i.e. $|T_1 - \langle T_2^1 \rangle| < 1.5$ in all 40 simulations, is defined as the synchronization window.

Results

Comparison of synchronization windows of different types of STDP

We investigate the width of synchronization window of various STDP curves, with the same set of parameters. The period of the pre-synaptic neuron is chosen to be 171ms, which falls into the range of theta waves. Several reasons make us choose such a long period. First, it has fairly wide synchronization windows which allow comparisons in a relative precise manner and can provide clearer information about synchronization windows of various STDP learning rules. Second, the slow theta waves always involve many neurons that fire synchronously (Bilkey and Heinemann 1999; Pape, Narayanan et al. 2005). Also, theta waves have many interesting implications. For example, theta waves are normally absent in healthy awake adults, but appear during the state of meditation (Aftanas and Golosheykin 2005). During emotional arousal and various types of rhythmic activities during sleep neurons in the amygdala produce theta activity (Paré and Collins 2000; Paré, Collins et al. 2002). And it is known that coherent theta activity (4-8Hz) in amygdala-hippocampal circuits is deeply involved in fear memory (Seidenbecher, Laxmi et al. 2003).

With fixed period of the pre-synaptic neuron T_1, we evaluate the coupled period of postsynaptic neuron $T_2{}^1$ when it is driven by the pre-synaptic neuron. The values of learning parameters used in c-STDP, dc-STDP, dc-aSTDP are $A_{plus}=9nS$, $A_{sub}=6nS$, $t_{plus}=100ms$, $t_{sub}=200ms$, *additionally* $\tau_0 = 30ms$ *in c-STDP,* and in in-STDP ar*e* $A_{plus}=A_{sub}=8nS$, $t_{plus}=100ms$, $t_{sub}=200ms$. Our model and most values of parameters are from the model of Nowotny for c-STDP, expcept A_{plus}, t_{syn}, Vs_{lope}, g_{max} are different(Nowotny, Zhigulin et al. 2003). Especially, T_1 is fixed at 171*ms* in our simulations while T_2 is set to constant value 300*ms* of Nowotny's work.

The window of synchronization (upper panel) and quality (middle panel) of dc-STDP are presented in Fig.2 as an example. We scan T_2 from 150ms to 320ms. The upper panel shows the number of synchronization times in 40 simulations, for each T_2. It is clear that, in certain range of T_2, simulations from different initial values may have different results. Only when T_2 falls into the segment from 194 to 221, the post-synaptic neuron can synchronize with the pre-synaptic neuron from any initial value. It is easily found that there are some T_2 corresponding to the number of synchronization times between 1 and 39. In this situation, we present the three possible states of post-synaptic neuron's firing in Fig.2 (lower panel)--keeping the initial period(squares), oscillating(circles), and synchronizing with the pre-synaptic neuron(dots). Obviously, these states are independent of the synchronization criteria we set.

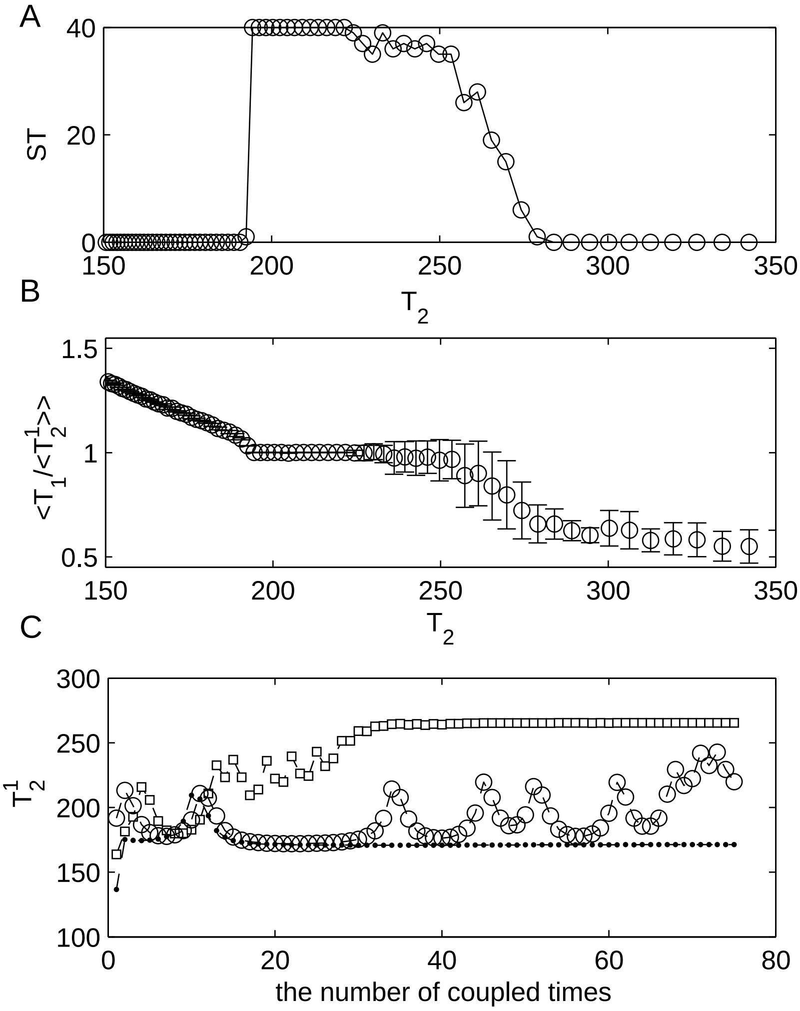

Figure 2: Synchronization results for dc-STDP rule carrying out 40 times. Fig.2A shows the number of synchronization times(ST) of fixed T_2 varying from 150 *ms* to 320*ms*. Synchronization window is from 194 to 221. The probabilistic synchronization window is from 222 to 289. Fig.2B presents the quality of synchronization against the ratios of uncoupled periods. Fig.2C, we fixes T_2=265*ms* which falls into the probabilistic synchronization window. There are three states of the coupled period of post-synaptic neuron $T_2{}^1$ when we carry out 40 stimulation times: (i) keeping the initial period(squares); (ii)oscillating(circles); (iii) synchronizing with the pre-synaptic neuron(dots).

For the purpose of discussing the function role of STDP rules in synchronization, the synchronization windows of various type of STDP are plotted in Fig.3, in which the case of constant synaptic conductance is also included as a comparative tool. The same parameter values used in simulations ensure a fair comparison. In our simulation studies, the synapse strength is between *0nS* and *25nS*. We choose the maximal synaptic strength and the middle synaptic strength of STDP synapse as the synaptic strength of constant synapse in this study. Interestingly, because the synchronization windows for STDP rules are narrower than synchronization window for constant synapse $g=25nS$ in Fig.3, these results, opposite to previous reports, indicate that all STDP rules don’t enhance synchronization comparing with the constant synapse under the chosen parameters.

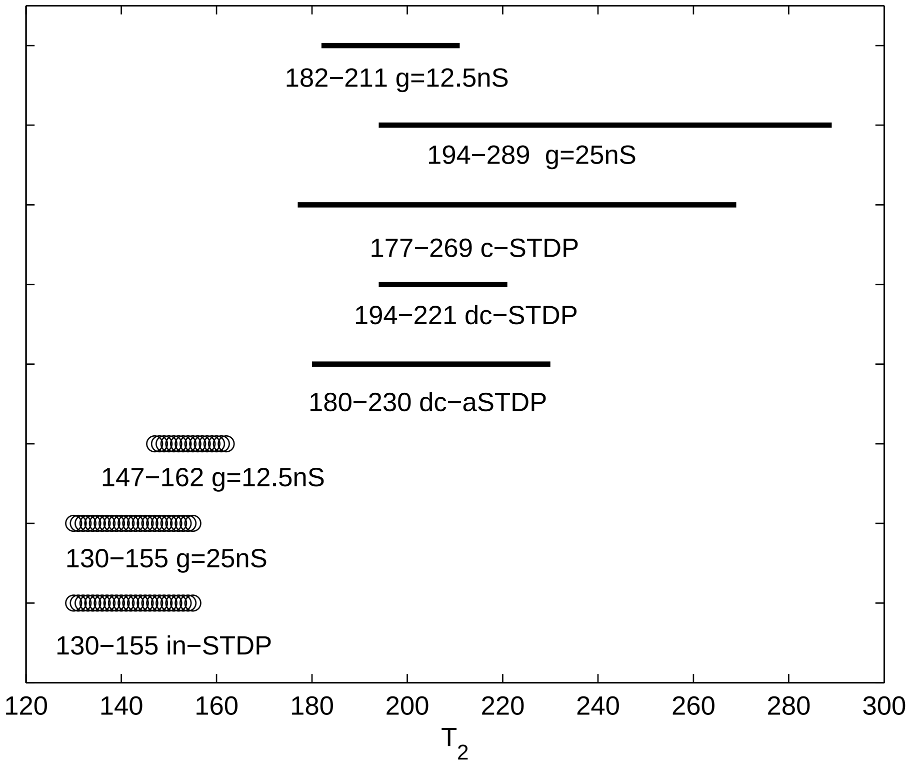

Figure 3: The synchronization window of different types of learning curves. The top five lines (points) are excitatory synapses and the bottom three lines (circles) are inhibitory synapses. See legends in the figure.

Several other points are worthy of detailed describing. First, increasing the excitatory constant synaptic connections from *12.5nS* to *25nS*, leads to a wider synchronization window. However, the wider window could not totally contain the smaller one. It extends towards larger T_2, but loses a portion of smaller T_2. The case of increasing inhibitory constant synaptic connections is alike. Second, the widest range of synchronization window are achieved by the excitatory constant connection *g=25nS* and c-STDP rules. However, the lower boundary of the synchronization window of c-STDP is much nearer to T_1 than that of excitatory constant synapse.

Thirdly, although c-STDP and dc-STDP are fitted from the same set of experimental data, c-STDP has a much wider synchronization window than dc-STDP.

We conclude that all STDP don't give rise to enhanced synchronization and the window of dc-STDP is surprisingly narrow under the chosen parameters in Fig.3. Therefore, the questions about what bring about these results inspire us to study further. At the same time, Fig.2 shows a large part of probabilistic synchronization, whose range is a subset of the synchronization window of constant synaptic connection with *g=25nS*. We are interested in if the probabilistic synchronization could be enhanced into absolute synchronization by modulation of learning curves. These are the theme of the next section.

The effect of learning parameters on synchronization

In order to establish the functional role of STDP clearly, we consider if the learning parameters for each STDP rule have important effect on synchronization. In addition, synchrony-asynchrony transition plays important role in the brain. An increase in the degree of synchrony of a uniform input can cause transitions between memorized activity patterns in the order presented during learning. However, if synchronous input is at a low level, transitions can't occur (Aoki and Aoyagi 2007). The synchrony-asynchrony transition have also been implemented in controlling winner-take-all competition (Lumer 2000), the next recalled time of associative memory (Aoyagi and Aoki 2004) and the fine structure of cell assemblies (Akimitsu, Okabe et al. 2007).

In this section, we will discuss the flexibility of the synchronization window, by exploring regulation of width of the synchronization window， whose boundary indicates the synchrony-asynchrony transition. We take the modulation of learning parameters as the method to regulate the synchronization window. There are four parameters that determine a learning rule A_{plus}, A_{sub}, t_{plus}, t_{sub} .With three of them fixed and only one parameter changing, we could explore its influence on the width of the synchronization window. For example, A_{plus} increases from an adequately small value 1*nS* to *20nS* with a step of 1*nS*. We have also scanned values that are beyond *20nS*, but find that the effect of increasing A_{plus} is saturated around *20nS*. Further increasing A_{plus} brings no more effect. Other parameters are fixed as: $A_{sub}=6nS$, $t_{plus}=100ms$, $t_{sub}=200ms$, $T_2 =233ms$. In these conditions, we present the effect of A_{plus} on the location of the synchronization window with the dc-STDP rule. We carry out simulations 40 times, each from different initial values.

Fig.4A shows the value of ARP (average change of relative period ratio) = $<T_2 -< T_2^1 >>/(T_2 -T_1)$ for different A_{plus}. Some points have value 0 or 1, which means post-synaptic neuron keeping initial period or achieving synchronization with the pre-synaptic one, respectively. Some points have values other than 0 or 1. Fig.4B gives an explanation that these points correspond to probabilistic synchronizations with fixed $T_2=233$.

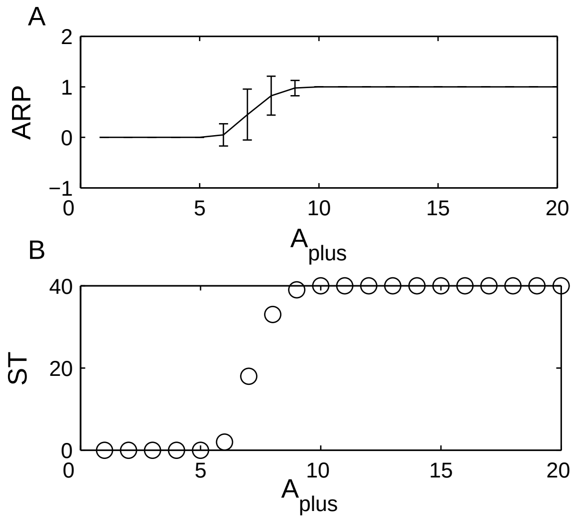

Figure 4: It shows that the regulation of A_{plus} to synchronization for the dc-STDP carrying out 40 times of stimulations. $A_{sub}=6nS$, $t_{plus}=100ms$,$t_{sub}=200ms$, $T_2 =233ms$.Top: The value of ARP=$<T_2 -< T_2^1 >>/(T_2 -T_1)$ is 0 or 1 which respectively means post-synaptic neuron keeping initial period or achieving synchronization. Bottom: The number of synchronization times (ST) in 40 stimulations against A_{plus}.

We find that the absolute synchronization range is *10nS* to *20nS* in Fig.4B. According to the definition of synchronization window of T_2, we can similarly define *10nS* to *20nS* as the synchronization window of A_{plus}, with dc-STDP rule and other fixed parameters. The boundary of this synchronization window indicates where synchrony-asynchrony transition happens when changing A_{plus}.

From the results of Fig.4, the reason why synchronization window of constant synapse is wider than that of STDP rules (Fig.3) may be explained by learning parameters. To figure out a global picture of the effect of A_{plus} on synchronization for c-STDP rule, we then determine the synchronization window of A_{plus} with different T_2. A_{plus} increases from 1*nS* to *20nS* with a step of 1*nS* while other three parameters keep initial values: $A_{sub}=6nS$, $t_{plus}=100ms$, $t_{sub}=200ms$. We choose some typical values of T_2 to character the global picture. The points in Fig.5A show the synchronization range of A_{plus} with T_1 divided by the chosen values of T_2. The lower boundaries, as well as those upper boundaries that are other than *20nS*, indicate the position of synchrony-asynchrony transitions. For example, when T_2 is 177*ms* equivalent to $T_1/T_2=0.966$, A_{plus} outside of the points range from 8*nS* to 11*nS* can not lead to synchronization between the two neurons. In addition, for those points marked on the horizontal axis, synchronization could not be established no matter what values A_{plus} take.

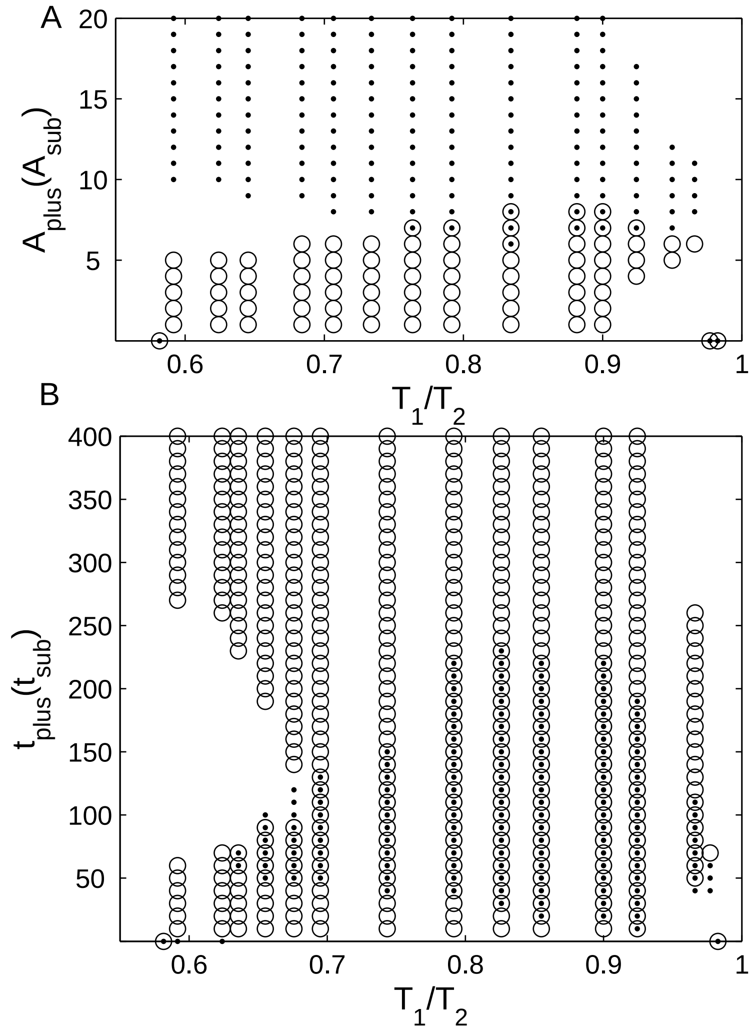

Figure5: The range of learning parameters leading to synchronization for all 40 stimulations for c-STDP is presented. The points are the range of A_{plus} and t_{plus} which can make neuron synchronization. The circles are the range of A_{sub} and t_{sub}. We choose some values of T_2 to investigate the effect of A_{plus}, A_{sub}, t_{plus} and t_{sub} on global synchronization. A_{plus} and A_{sub} vary from 1*nS* to 20*nS*. Let t_{plus} and t_{sub} vary from 10*ms* to 400*ms*. There are some values of T_1/T_2 marked on the horizontal axis can't be entrained to achieve synchronization in Fig5.A and Fig5.B.

According to this global picture, the intersection of those synchronization ranges of A_{plus}, which is from *10nS* to *11nS*, identifies the range of A_{plus} that would lead to the widest synchronization window which is from 177 to 289. This optimal synchronization window for c-STDP is wider than constant synapse $g=25nS$. Thus, the learning parameters strongly influence the role of STDP on synchronization.

Taking the same method as A_{plus}, we study the effects of other three parameters on synchronization. Fig.5 presents the situation of A_{sub} varying from 1*nS* to 20*nS* with the step 1*nS* (circles, Fig.5A), t_{plus} (points, Fig.5B) and t_{sub} (circles, Fig.5B) both varying from 10*ms* to 400*ms* with the step 10*ms*. When change one parameter to explore how the range of synchrony evolves with T_2, other three parameters keep their initial values as in Fig.3. Similarly, the global modulation picture of four learning parameters for other learning rules can be got. We only give the situation for c-STDP rule in Fig.5.

The optimal synchronization windows for various learning rules are presented in Fig.6 comparing with previous synchronization windows obtained in Fig3. The parameters used for optimal synchronization windows are presented in Table 1. The parameters are derived according to regulating one parameter while other three parameters keep initial values. Under the optimal parameters, the synchronization windows for STDP rules are not narrower than constant synapse $g=25nS$. Consequently, the reported important role of STDP in synchronization should be dependent on learning parameters.

Table 1: The parameters for optimal synchronization windows

STDP rule	A_{plus} (nS)	A_{sub} (nS)	t_{plus} (ms)	t_{sub} (ms)
c-STDP	9	6	100	270
dc-STDP	9	6	100	50
dc-aSTDP	9	6	100	350
in-STDP	8	8	100	200

For the first three STDP rules, the optimal synchronization windows are got by regulating t_{sub} while other three fixed learning parameters keep initial values. For the last STDP rule, the initial values are the optimal parameters.

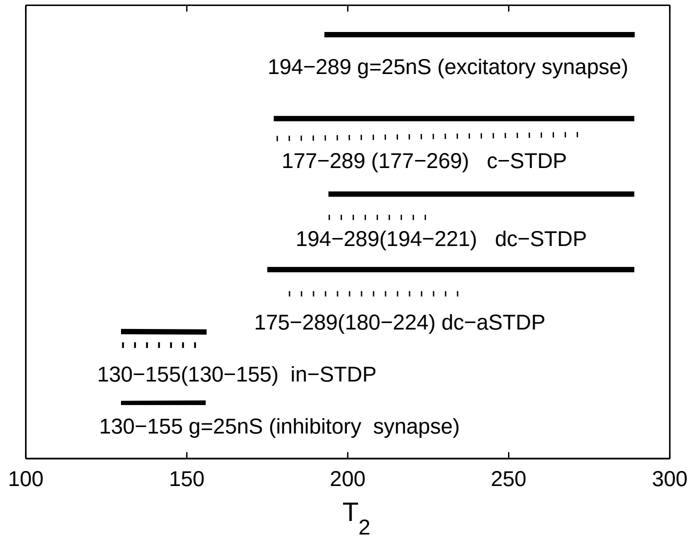

Figure 6: We present the optimal synchronization window for various learning rules compared with the Fig.3 showing. The top and the bottom lines of this figure are the synchronization windows of excitatory and inhibitory constant synapse respectively. In addition, there are four pair lines in the middle panel for STDP rules. Each pair includes optimal synchronization window (top line) and previous synchronization window in Fig.3 (bottom line). The left range is optimal synchronization range and the range in round bracket is previous synchronization range.

The synchronization mechanism

It is important to understand the properties which neural synchronization depends on. We take into account this problem from two aspects.

On the one hand, synchronization correlates with the chosen STDP rules. For excitatory synapse, we find that the optimal synchronization windows for c-STDP rule and dc-aSTDP rule are almost equal, and are wider than constant synapses. However, comparing with these two rules, the optimal synchronization window for dc-STDP rule is much narrower. For inhibitory STDP, the synchronization window is the same as that for constant synapse under connection strength $g=25nS$.

Accordingly, what makes the optimal synchronization windows for various STDP rules different deserves an explanation. The stationary synaptic conductance is a necessary condition for stationary synchronized state (Nowotny, Zhigulin et al. 2003) We find that the mechanisms of synchronization caused by c-STDP (or dc-aSTDP) and dc-STDP (or in-STDP) are different in our model.

Fig.7A and Fig.7B show the average $\triangle t=t_{postspike}- t_{prespike}$, and synaptic strength after an episode of coupling time for c-STDP rule. Parameters in Fig.7 are same as those in Fig.3.There are two types of behavior for $\triangle t$ when synchronization occurs (Fig.7A). In a section of constant $\triangle t$, the synaptic strength doesn’t achieve the maximal value. Apparently, in this situation, postsynaptic neuron achieves synchronization with the pre-synaptic neuron depending on the balance between potentiation and depression of synaptic conductance. In the rest part of synchronization window, the synaptic strength achieves the maximal value (Fig.7B). It indicates that for larger $T_{2,}$ postsynaptic neuron achieves synchronization depending on the effect of maximal synaptic conductance. For dc-aSTDP rule, the synchronization mechanisms are similar to c-STDP rule. When the post-synaptic neuron synchronizes with the pre-synaptic neuron under small $T_{2,}$ the change of synaptic potentiation and depression cancel each other. However, for the small portion of synchronization window at the right side, synaptic conductance gets the maximum at the stationary synchronized state.

For dc-STDP and in-STDP rules, the synchronization mechanisms may be different with the above two STDP rules. Because when the potentiation and depression of synaptic conductance cancel each other, △t must be a fixed value for the selected STDP rule. However, for the dc-STDP rule (Fig.7C and 7D), △t keeps varying which means that the potentiation and depression of synaptic conductance don’t cancel each other at the synchronization state. But, it is easily found that the synaptic conductance is at the stationary maximum for dc-STDP rule. Thus, postsynaptic neuron achieves synchronization completely depending on the effect of the maximal synaptic conductance for dc-STDP rule. The state of in-STDP rule is similar to dc-STDP rule.

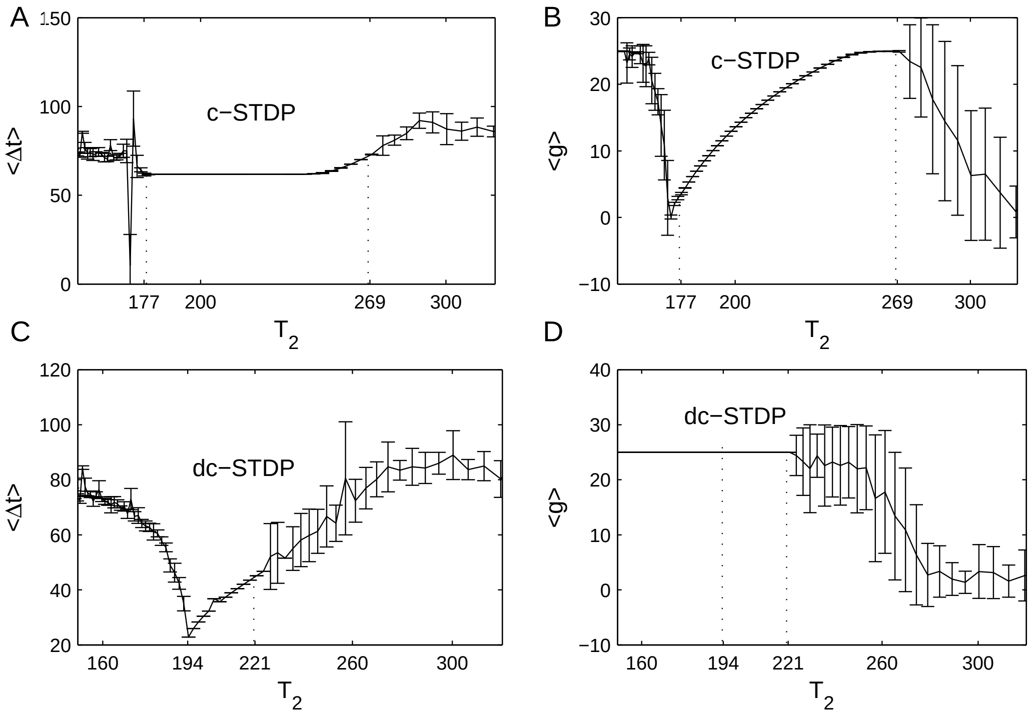

Figure7: Five pictures are obtained from the same values of parameters and model of Fig.3. The front four pictures take the function of tanh to limit synaptic strength for c-STDP and dc-STDP rules. Fig.A and C show the average spike time interval of post-neuron's and pre-neuron's spike time for c-STDP and dc-STDP over some time after a period time of coupling respectively. Fig.B and D present the average synaptic strength for c-STDP and dc-STDP rule respectively. Each subplot has two dash lines what indicate the boundary of synchronization window.

As a result, neural synchronization mechanism can be different for various STDP rules. For the few T_2 at the right side of the synchronization window, the synaptic conductance achieves the maximum with c-STDP and dc-aSTDP rules. This result is obvious. Because the frequency mismatch is larger, the synapse needs to be stronger to entrain the post-synaptic neuron. But for most part of synchronization window at the left, c-STDP and dc-aSTDP rules rely on the balance of potentiation and depression. Instead of balancing out potentiation and depression during one cycle, dc-STDP and in-STDP rules depend on synaptic strength achieving its maximum. It is important to understand why c-STDP and dc-aSTDP rely on the balance of potentiation and depression while dc-STDP and in-STDP don't.

Nowotny et al. have introduced the mechanisms behind the enhancement of neural synchronization by c-STDP rule which rely on the balance of potentiation and depression. The synapse strength remains stable regardless of postsynaptic neuron firing later or earlier attributed to the specific shape of c-STDP curve. The situation of c-STDP is similar to dc-aSTDP. We adopt the similar analysis method (Nowotny, Zhigulin et al. 2003) here for the dc-aSTDP and dc-STDP. The time lags are recorded as $\triangle t_1$ and $\triangle t_2$, where $\triangle t_1 + \triangle t_2 = T_1 = T_2^1$ and $\triangle g_1 - \triangle g_2 = 0$ (Fig.8) at this state. If post-synaptic neuron fires faster, $\triangle t_1$ becomes smaller. Synaptic strength will be depressed, due to $\triangle g_1 - \triangle g_2 < 0$ for dc-aSTDP rule, so that the post-synaptic neuron is less excited and goes back into the synchronized state (Fig.8B). The other direction can be analyzed in the same way for dc-aSTDP. But for dc-STDP, when post-synaptic neuron fires faster, synaptic strength will be increased due to $\triangle g_1 - \triangle g_2 > 0$ in this case. The post-synaptic neuron is more excited and can't go back into the synchronized state (Fig. 8C). The opposite direction is the same case for dc-STDP and can't go back into the synchronized state. Therefore, the synchronization mechanisms between these two rules are different.

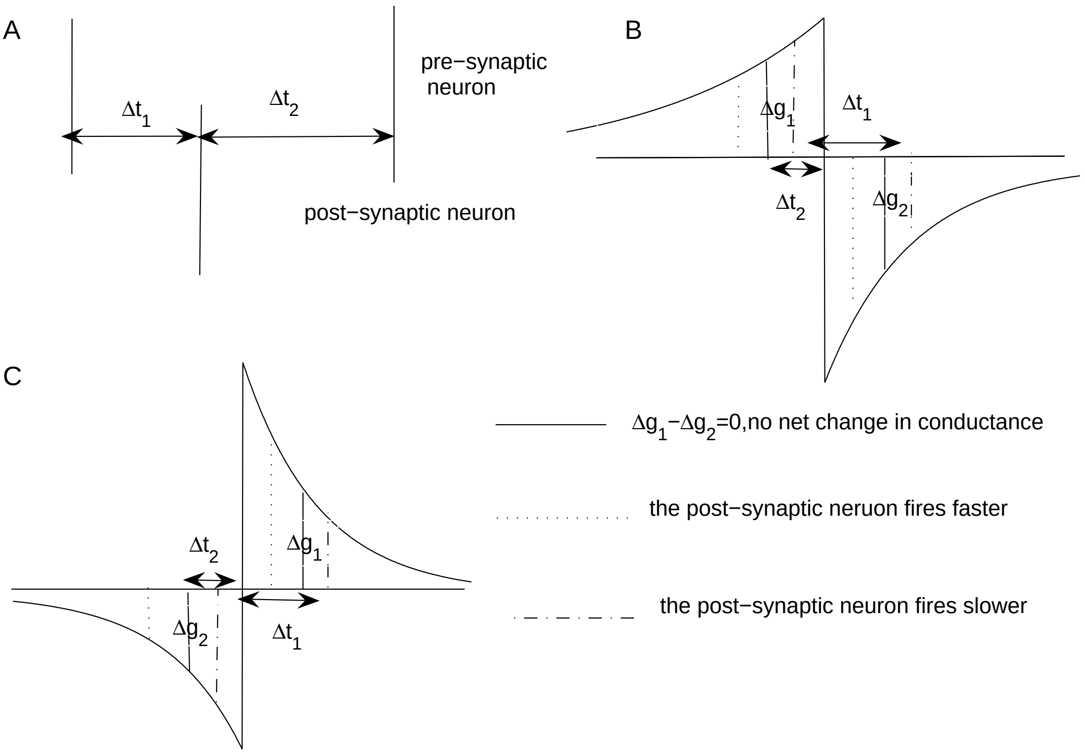

Figure8: Different synchronization mechanisms for different STDP rules. A shows the situation of $\triangle t_1 - \triangle t_2 = T_1 = T_2$, where $\triangle g_1 - \triangle g_2 = 0$(the solid lines in B and C). B and C present the change of synaptic strength of dc-aSTDP and dc-STDP rules respectively.

For in-STDP rule, we can easily find that $\triangle g_1 - \triangle g_2$ is always positive, where $\triangle g_1 - \triangle g_2 = A_{plus} *(\exp(-x/t_{plus})-0.5)- A_{sub} *(\exp(x-171/t_{sub})-0.5)$ and values of parameters are the same as Fig. 3. It means that the potentiation and depression of synaptic conductance during one period can’t achieve balance. The synaptic strength must achieve the maximum resulting from $\triangle g_1 - \triangle g_2 > 0$.

On the other hand, learning parameters also play important role in neural synchronization. We try to explain the role of a learning parameter by considering how it influences the synapse conductance, which is a major factor for synchronizing neurons with a given mismatch of intrinsic frequencies.

It is obvious that if synaptic conductance becomes stronger, it can make larger T_2 achieve the same period with pre-synaptic neuron for various STDP rules. Thus, with other three learning parameters fixed, larger A_{plus} values, which are corresponding to the stronger stable synaptic strength for c-STDP rule, can cause larger T_2 synchronization. Similarly, smaller A_{sub} values will give rise to larger T_2 synchronization for c-STDP (Fig.5A). Moreover, from Fig.5B, moderate t_{plus} values can also make larger T_2 synchronized to T_1 because these values bring about stronger synaptic strength. We can prove this perspective by simple calculus reasoning. Based on the expression of c-STDP rule in section 2, let t_{plus} be variable and other parameters keep constant. $\triangle g_{raw}$ is viewed as the function of variable t_{plus}. By calculating the derivative of $\triangle g_{raw}$, we can find that $\triangle g_{raw}$ is a first increasing and then decreasing function when t_{plus} increases gradually. Thus, medial values of t_{plus} can result in stronger synaptic strength. By the same reasoning for t_{sub}, we can conclude that smaller or larger t_{sub} can make larger T_2 synchronization for c-STDP (Fig.5B).

The effects of learning parameters on synchronization about other learning rules are similar to the c-STDP rule for larger T_2. However, smaller T_2 values leading to synchrony are only related to c-STDP and dc-aSTDP, because they rely on the balance of depression and potentiation which could lead to an appropriate low stable synaptic strength. Therefore, only proper learning parameters got by regulating the effect of learning parameters on synchronization are required for smaller T_2 achieved synchronization.

Finally, we conclude why the widest synchronization windows for some STDP rules are different. From the above statements, we find that two aspects affect synchronization. One is the maximum synaptic strength which can make larger T_2 synchronize. The other is the balance of depression and potentiation which can make smaller and moderate T_2 synchronize. For c-STDP and dc-aSTDP, they can achieve the widest synchronization windows through modulating the two aspects. However, dc-stdp and in-stdp rules, due to their specific shape, can only make use of the first one. This implies that the widest synchronization windows for dc-STDP and in-STDP rules can’t exceed the synchronization windows for the maximum constant connection strength under the same model respectively. Therefore, the optimal synchronization windows for c-STDP and dc-aSTDP are wider than those for dc-STDP and in-STDP.

Conclusion

STDP plays important functional role in neural synchronization. The mechanism of STDP in neuronal synchronization is still not completely clear. Inspired by previous experiments and theoretical researches, we study the important aspects of STDP induced synchronization in this paper, such as the role of various STDP rule in synchronization, the widest synchronization window through regulating learning parameters, and synchronization mechanism.

In order to explore the functional role of STDP in synchronization, we compare synchronization windows of different types of STDP rules with that of constant synapse under the same model parameters. For the given parameters, not all synchronization windows are enhanced by STDP rules.

Synchronized responses have a stronger influence on cells at subsequent processing stages than non-synchronized responses (Alonso, Usrey et al. 1996; Brecht, Singer et al. 1998; Singer and Strategies 1999). And the enhanced precise synchronization is important in improving a rapid and reliable transmission of information about sensory changes (Diesmann, Gewaltig et al. 1999; Womelsdorf, Fries et al. 2006). Recent researches have reported various methods to enhance synchronization, such as, selective attention (Fries, Womelsdorf et al. 2008) and time delay (Qing-Yun and Qi-Shao 2005). Here, we present the effect of modulation of learning parameters on synchronization and optimal synchronization window which are not narrower than constant synapse. The optimal synchronization windows by c-STDP and dc-aSTDP rules are much wider than constant synapse. It indicates that the function role of STDP rule in synchronization depends on the learning parameters.

The synchronization mechanism is also described here. Different shape of STDP rule can cause different optimal synchronization window. The optimal synchronization windows of c-STDP and dc-aSTDP are wider than that of dc-STDP for excitatory synapse. For c-STDP rule and dc-aSTDP rule, a stationary synchronized state completely depends on the balance between potentiation and depression or the maximal synaptic conductance. However, for dc-STDP and in-STDP rule, the stable synchronized state depends on the maximal synaptic conductance under the self-limitation of synaptic strength. If we change the type of bound of synaptic strength for dc-STDP rule from self-limitation to artificial bounds, we find that the synchronization mechanism doesn't change. In a word, on the one hand, the synchronization range of dc-aSTDP and c-STDP can achieve the optimal synchronization window of dc-STDP, depending on the maximal synaptic conductance. On the other hand dc-aSTDP and c-STDP can extend the synchronization windows to include smaller T_2 by the balance between potentiation and depression.

The firing pattern of neurons is regular in this paper. Many neurons in brain areas present regular firing. For example, neurons in cat area 17 can be grouped in 4 different electrophysiological cell classes， including regular spiking (Nowak, Sanchez-Vives et al. 2008). And spontaneous, regular action potentials were observed both with cell-attached patch recordings as well as with whole cell current-clamp recordings for cholinergic neurons in the parabigeminal nucleus of the rat midbrain (Goddard, Knudsen et al. 2007). Neuronal synchronization properties with regular firing neurons have been studied. For example, whether pyramidal neurons in different cortical layers exhibit similar tendencies to synchronize are studied (Tsubo, Takada et al. 2007). Based on this point and the functional role of STDP in synchronization, we explore the synchronization windows of various STDP rules from the view of neurons' regular firing.

STDP-mediated synchronization is a remarkably robust phenomenon against strong noise (Nowotny, Zhigulin et al. 2003; Zhigulin, Rabinovich et al. 2003). Although our simulation isn't under the noise environment, our results may represent some predictions for STDP-mediated synchronization in noisy environment. In addition, from Fig.2A, we can clearly see that some synchronization number is between 1 and 39. We estimate that it may be related to phase.

Our results present that the range of T_2 values leading to synchrony increases strongly if the constant synaptic connection is increased from 12.5*nS* to 25*nS*. Nowotny et al. find that the extent of synchrony does not change considerably by doubling the synaptic conductance(Nowotny, Zhigulin et al. 2003). Their result is not conflict with our result. There are three parameters are different, $V_{slope,}$ $t_{syn,}$ and $g_{max.}$ Furthermore, T_1 is fixed at 171*ms* in our simulation while T_2 is fixed at 300*ms* in theirs. When T_2 is fixed, the range of T_1 values leading to synchrony is limited from 0 to T_2 no matter how the strength of constant connection changes. However, when T_1 is fixed, the range of T_2 values leading to synchrony can vary from T_1 to very large value due to the increase of the constant connection. At the same time, if we adopt the same parameters with Nowotny's paper, the similar result can be got. Furthermore, when constant synaptic connection is 0*nS*, it is clear that two neurons with different initial periods can't synchronize. This situation means that the length of synchronization window is 0. Along with the increase of strength of constant connection, some T_2 must cause the synchronization. Thus, it is easily found that synchronization window must become wider by increasing the constant connection to some degree.

We mainly discuss synchronization for different STDP rules in this paper. The question of how the time windows of various STDP rules are biophysically regulated remains relatively unexplored. There are some experiments using neuromodulators to study the time window for STDP (Seol, Ziburkus et al. 2007). We are interested in building molecular kinetic equations for STDP to explain our results.

It has been proposed that conscious perception depends on the transient synchronization of widely distributed neural assemblies (Thompson and Varela 2001). And long-distance synchronization plays a role in triggering the cognitive processes associated with conscious awareness (Dehaene, Changeux et al. 2006). The changed learning parameters by neuromodulators may influence the cognitive processes. In addition, some diseases and the function of brain are related with synchronization mentioned above, especially in theta(4-8Hz) rhythm synchronization during fear memory retrieval (Seidenbecher, Laxmi et al. 2003) which is consistent with what we considered here. Therefore, our work may advance understanding of synchronization to some extent. And we expect our simulation results will provide some help for related diseases treatment.

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