Contextual, one-sector, non-regular fuzzy model based on 4 knowledge points

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Abstract

The paper presents a method how to construct a non-regular fuzzy model based on 4 points of expert knowledge. Non-regular fuzzy models considerably differ from regular ones, which are based on the regular, rectangular partition of the input space. They allow for considerable decreasing the rule number and thus for constructing sparse models and for overcoming the phenomenon called “curse of dimensionality”. Non-regular fuzzy modeling is rather not possible without a new coordinate system, which was called contextual, non-parallel coordinate system that also is described in the paper. The non-regular modeling method was illustrated by an example.

Keywords: fuzzy modeling, fuzzy logic, non-regular models, coordinate systems, fuzzy reasoning.

Introduction

Methods of fuzzy modeling were described in many books, e.g. in [3,4,5,6,7,10]. Present methods are based on regular, rectangular partition of the system input space, Fig.1. Such partition is used by people in their mental modeling of dependences $y=f(x_1,\ldots, x_n)$ observed in the surrounding world. The regular, rectangular partition facilitates understanding and keeping in mind recognized dependences.

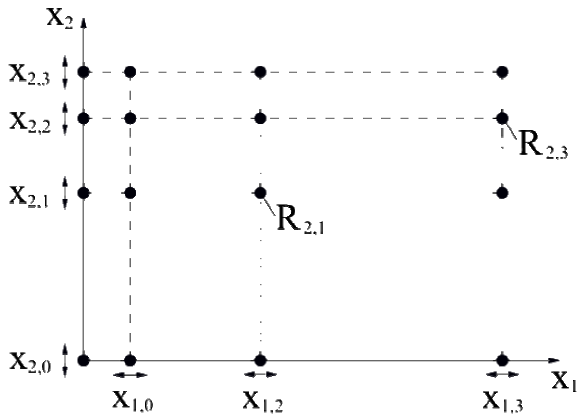


Fig.1. Regular, rectangular partition of the input space in the case of 3D-dependence $y=f(x_1,x_2)$.

Regular models have a large number of nodes. Each node has to be defined by one rule $R_{i,j}$ that represents at this node the dependence under modeling. Mostly the Mamdani type (1) of rules is used.

$$R_{i,j}\text{: IF}(x_1 \text{ close to } x_{1,i}) \text{ AND } (x_2 \text{ close to } x_{2,j}) \text{ THEN } (y \text{ close to } y_{i,j}) \tag{1}$$

Linguistic quantifiers as e.g. "close to $x_{1,i}$" are defined by membership functions. Regular fuzzy models (shortly RF-models) apart from their good points also have weak points. Below 3 of them are given.

1. Together with increasing number of quantifiers defining particular variables x_1, x_2, ..., x_n, y and of the number n of input variables the number of rules, which are required in a RF-model, is increasing in the avalanche way. This phenomenon was called "curse of dimensionality" [2]. It hinders modeling of high-dimensional problems or even makes it impossible.
2. At tuning the RF-model, e.g. in form of a neurofuzzy network, change in position of one, single quantifier as "close to $x_{1,i}$" influences not one but simultaneously many rules which contain this quantifier in their premises. In the consequence, the RF-model improves its accuracy in one region but makes it worse in other regions of the input space. It considerably hinders modeling [5,6,7,10].
3. The input space X_1*X_2*…*X_n of real systems usually are not rectangular but are of other non-regular shapes, e.g. of elliptic one. Therefore the rectangular partition frequently is not the optimal one [9].

To overcome or at least to diminish the “curse of dimensionality” scientists proposed certain means, e.g. the non-grid input space partition [2]. Fig.2 presents 2 such partitions together with the non-regular partition proposed by the authors.

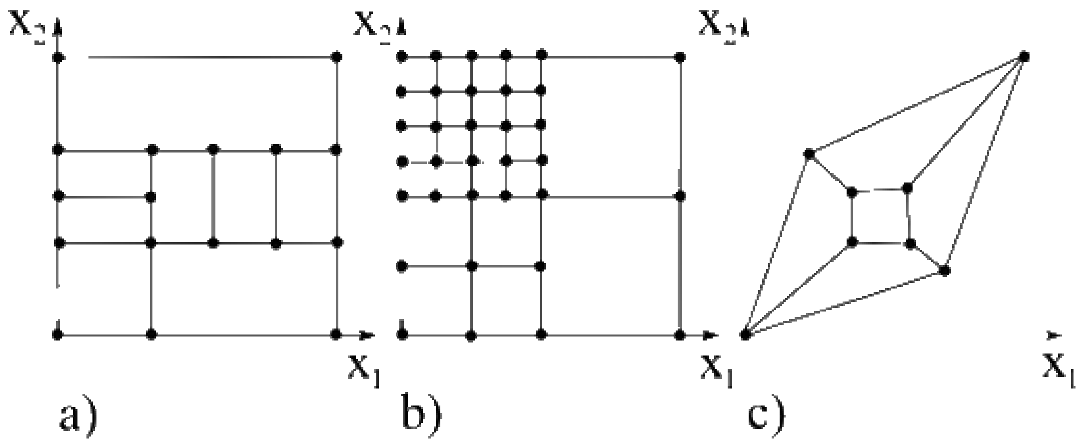


Fig.2. Two examples of the non-grid input space partitions: a)rectangular, non-grid partition, b) square non-grid partition, and c) example of a non-regular, non-grid partition proposed in the paper.

The rectangular and square non-grid partition does not fully solve the problem of the “curse of dimensionality” and certain weak points of RF-models are not removed. Instead, the non-regular partition (Fig. 2c) gives possibility to fully overcome this phenomenon or to considerably decrease the number of model rules and to construct sparse fuzzy models. In the sequence the idea of the non-regular fuzzy modeling will be presented. The author of it is Andrzej Piegat. The computer experiment shown in the paper was realized by Marcin Olchowy.

Contextual, non-regular, non-parallel coordinate system (CNRNP-coordinate system)

The non-regular fuzzy model (NRF-model) can in the general case consists of many type of sectors, e.g. of triangle sectors, of tetragonal sectors, of pentagonal sectors, of hexagonal sectors, etc. In this paper, because of its volume limitation only a NRF-model of the tetragonal sector will be shown. However, this model is very important because it is the basis for models of higher-order sectors.. Calculations in the NRF-model can be realized independently in each sector, Fig.2c. Particular model sectors have to adjoin at their borders. To facilitate the sector merging and to secure the unique calculation of the model output-value by neighbor sectors the linear interpolation at sector borders was assumed. However, inside of sectors the so called safe interpolation [8] is realized that generally is nonlinear one. The feature of the safe interpolation is possibly small model surface over the sector. In the NRF-model, in general case, calculations differ from those made in the RF-model.

To find a way of the NRF-model construction creation of a new coordinate system was necessary. It was called CNRNP-coordinate system. The idea of this system will be explained below. Let us assume, we have 4 points A,B,C,D of expert knowledge (2) about a dependence $y=f(x_1,x_2)$ existing in a system under consideration.

A: IF [(x_1,x_2) similar to (1, 9)] THEN (y similar to 5)
B: IF [(x_1,x_2) similar to (1, 4)] THEN (y similar to 1) (2)
C: IF [(x_1, x_2) similar to (6, 3)] THEN (y similar to 4)
D: IF [(x_1, x_2) similar to (13, 7)] THEN (y similar to 0)

Distribution of the expert knowledge points A,B,C,D is presented in Fig.3.

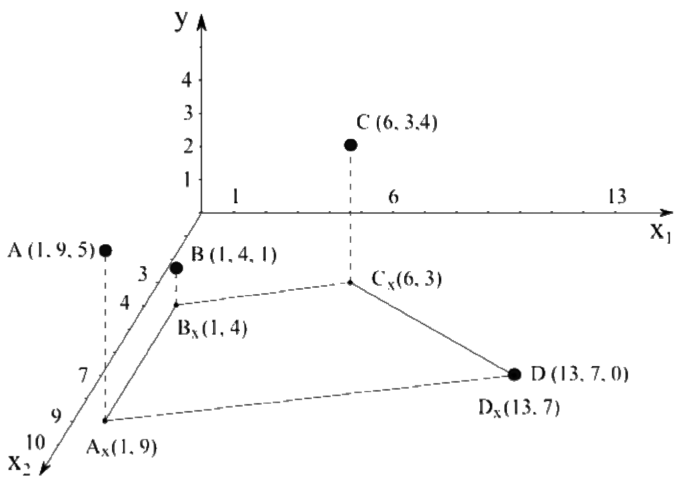


Fig.3. Non-regular distribution of the expert knowledge points A,B,C,D about a dependence $y=f(x_1, x_2)$ that create a context sector.

Fig.4 presents projections A_x, B_x, C_x, D_x of the knowledge points on the input space X_1*X_2 and also few values of the context coordinates α and β.

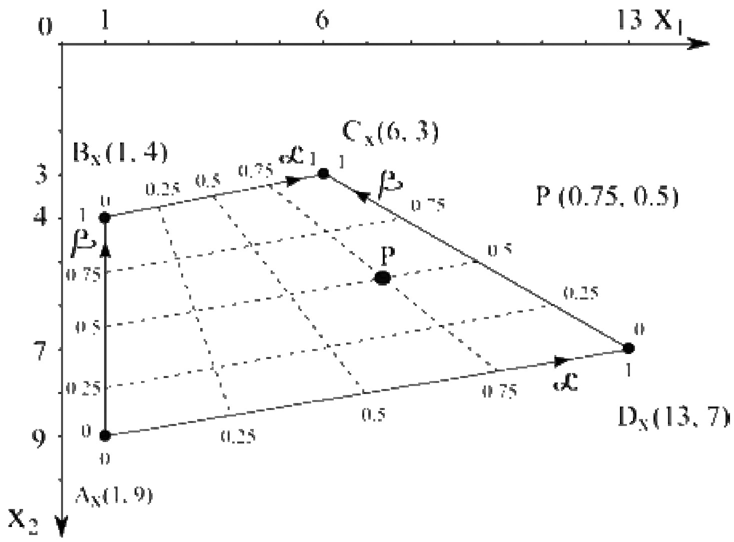


Fig.4. System of context coordinates α and β that corresponds to the points A_x, B_x, C_x, D_x of the expert knowledge.

It should be noted that values of the context coordinates are normalized to interval [0,1]. The main feature of the context coordinate system is proportionality of partition of sector borders. And thus the value α=0.5 lies in the middle of the border (A_x, D_x) and of (B_x, C_x). The value α=1/4 lies at the ¼ part of borders (A_x, D_x) and of (B_x, C_x) etc. Value α=0.75 can be interpreted as the contextual dissimilarity of point P lying inside the sector to the border (A_x, B_x) or more precisely, to the nearest point lying on this border. Value (1-α)=0.25 means the contextual similarity of the point P to the border (A_x, B_x). Appropriate meaning has the value of the coordinate β in relation to borders (A_x, D_x) and (B_x, C_x). Meaning of particular points of the context A, B, C, D is as follows: points B, C, D are negations (similarity equal to zero) of the point A, points A, C, D are negations of the point B, etc.

Task of any fuzzy model is delivering answers to questions as :

$$\text{What is the value of y if } [(x_1, x_2) \text{ is similar to } (4, 3)] \text{ ?} \qquad (3)$$

Because the question is formulated in the input space X_1*X_2 of the model it should be transformed in the space of α*β of the context variables. This transformation is based on notations presented in Fig.5.

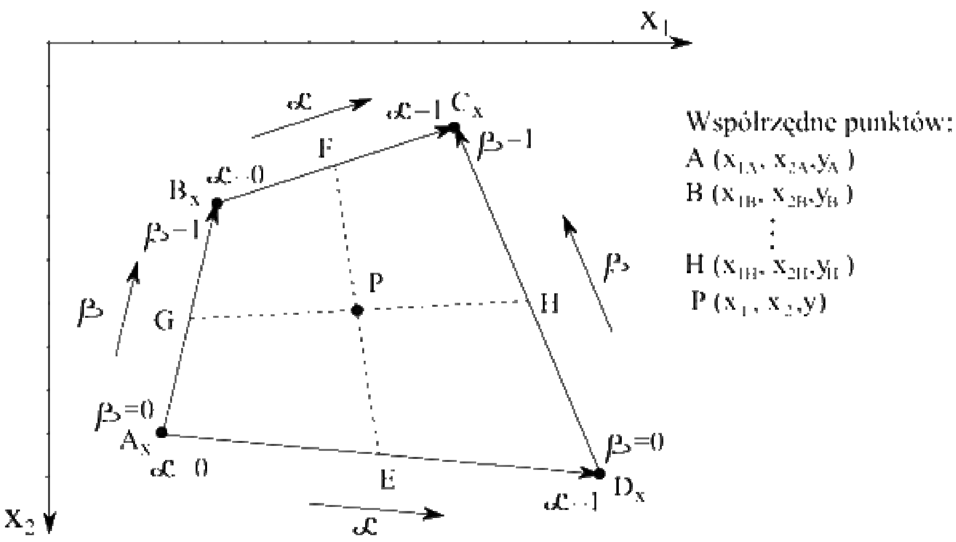


Fig.5. Notation assumed for transformation of Cartesian coordinates (x1,x2) in context coordinates (α, β).

Coordinates α of points F,P and E are equal, similarly coordinates β of points G,P,H (3).

$$\alpha_E = \frac{A_x E}{A_x D_x} = \alpha_P = \frac{GP}{GH} = \alpha_F = \frac{B_x F}{B_x C_x}$$

$$\beta_G = \frac{A_x G}{A_x B_x} = \beta_P = \frac{EP}{EF} = \beta_H = \frac{D_x H}{C_x D_x}$$

The following new notations will be used:

$a_1 = x_{1B} - x_{1A}$, $a_2 = x_{2B} - x_{2A}$, $a_3 = x_{1C} - x_{1D}$, $a_4 = x_{2C} - x_{2D}$, $a_5 = x_{1D} - x_{1A}$,
$a_6 = x_{2D} - x_{2A}$, $a_7 = x_{1C} - x_{1B}$, $a_8 = x_{2C} - x_{2B}$, $a_9 = x_{1A} - x_1$, $a_{10} = x_{2A} - x_2$, (4)

It should be noted that coefficients a9 and a10 are functions of x1 and x2. For points E,F,G,H from Fig.6 dependences (5) can be written.

$x_{1E} = x_{1A} + a_5\alpha$ $x_{1F} = x_{1B} + a_7\alpha$
$x_{2E} = x_{2A} + a_6\alpha$ $x_{2F} = x_{2B} + a_8\alpha$
(5)
$x_{1G} = x_{1A} + a_1\beta$ $x_{1H} = x_{1D} + a_3\beta$
$x_{2G} = x_{2A} + a_2\beta$ $x_{2H} = x_{2D} + a_4\beta$

For any point $P(x_1, x_2)$ lying inside the context equations (6) can be written.

$$x_1 = x_{1G} + \alpha (x_{1H} - x_{1G}) \qquad x_2 = x_{2G} + \alpha (x_{2H} - x_{2G}) \tag{6}$$

After appropriate transformation of equations (5) and (6) equations (7) are achieved that determine coordinates (α, β) corresponding to Cartesian coordinates (x_1, x_2).

$$\alpha\beta (a_3 - a_1) + a_5\alpha + a_1\beta + a_9 = 0 \qquad \alpha\beta (a_4 - a_2) + a_6\alpha + a_2\beta + a_{10} = 0 \tag{7}$$

Solution of equations (7) delivers formula for calculation of value of β.

$$K_1\beta^2 + K_2\beta + K_3 = 0 \tag{8}$$

where:

$K_1 = a_2a_3 - a_1a_4$

$K_2 = a_2a_5 - a_1a_6 + a_{10}(a_3 - a_1) - a_9(a_4 - a_2)$

$K_3 = a_5a_{10} - a_6a_9$

After the value of β for the point (x_1, x_2) is known then the α-value can be calculated from formula (9).

$$\alpha = \frac{a_1\beta + a_9}{(a_1 - a_3)\beta - a_5} \tag{9}$$

In certain cases, for certain distribution of the points A, B, C, D one or even two from the three coefficients K_1, K_2, K_3 can be equal to zero. Then solving equation (8) will be easier.

Example of a 1-sector, non-regular fuzzy model based on 4 knowledge points

There are given 4 points of expert knowledge (10).

A(1, 9, 5), B(1, 4, 1), C(6, 3, 4), D(13, 7, 0) (10)

The points and their projections A_x, B_x, C_x, D_x on the input space $X_1{*}X_2$ are shown in Fig.6.

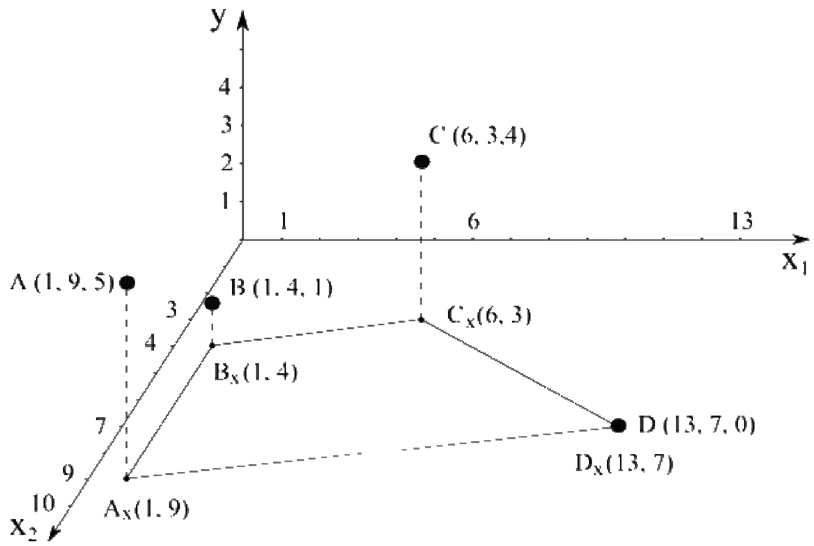


Fig.6. Points of expert knowledge about the dependence $y=f(x_1, x_2)$ in the space 3D.

The task is to construct a NRF-model capable of safe interpolation between the knowledge points and on its basis to calculate the output y-value for the question point P(5.25, 5.75).

Rules (11) of the model directly result from the knowledge points A, B, C, D.

R_A) IF [(x_1, x_2) similar to (1, 9)] THEN (y_A similar to 5)

R_B) IF [(x_1, x_2) similar to (1, 4)] THEN (y_B similar to 1)

R_C) IF [(x_1, x_2) similar to (6, 3)] THEN (y_C similar to 4) (11)

R_C) IF [(x_1, x_2) similar to (13, 7)] THEN (y_D similar to 0)

Functions as "similar to $(x_{1,i}, x_{2,j})$" are defined by equations (12). The notation as $\mu_{Ax}(\alpha,\beta)$ means the membership function of fuzzy set "similar to A_x), Fig.6. Formula (13) gives similarity functions of particular knowledge points.

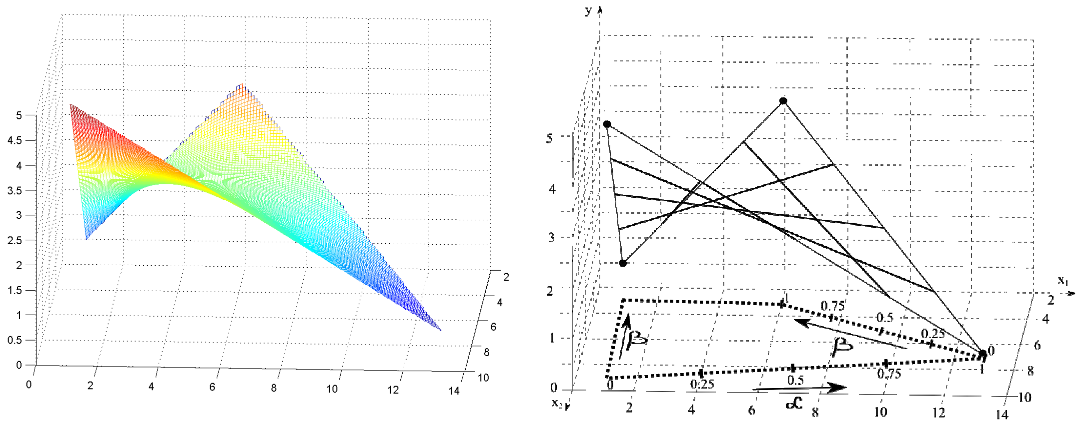
$$\mu_{Ax}= (1-\alpha)(1-\beta), \qquad \mu_{Bx}= (1- \alpha)\beta, \qquad \mu_{Cx}= \alpha\beta, \qquad \mu_{Dx}= \alpha(1-\beta) \qquad (13)$$

In the case of the question point P(5.25, 5.75), which is also the gravity center of the 4 points A, B, C, D calculations with formulas (8) and (9) give results: $\alpha=0.5$ and $\beta=0.5$. Calculations with formula (13) give results: $\mu_{Ax}= \mu_{Bx}=\mu_{Cx}=\mu_{Dx}=0.25$.

Basing on conclusions of rules from the rule base (11), with use of Mamdani-implication and of defuzzification with the center of gravity of singletons as optimal operations from the point of view of the safe interpolation [8] the y-value from formula (14) can be calculated.

$$Y(5.25, 5.75) = 0.25 \cdot 5 + 0.25 \cdot 1 + 0.25 \cdot 4 + 025 \cdot 0 = 2.5 \quad (14)$$

Full surface of the NRF-model is shown in Fig.7.



a) b)

Fig.7. Surface of the 1-sector fuzzy model based on 4 knowledge points A, B, C, D. Figure a) smooth surface, figure b) the surface shown with use of cuts for constant values of α and β. It should be noted that borders of the model surface from Fig.7 are, according to the earlier assumption, linear.

Fig.7b is particularly informative one. It shows that the nonlinear model surface consists (is constructed) of linear segments. Thus the surface is the maximally stretched one and it belongs to the smallest interpolation surfaces based of 4 given knowledge points and on linear borders. Thus it satisfies conditions of the safe interpolation explained in [8].

Conclusions

The paper presents a new method that enables creating non-regular fuzzy models based on 4 points of knowledge. Non-regular modeling is considerably more difficult than the regular one, however it opens new possibilities of construction and application of sparse fuzzy models with small number of rules at satisfactory precision of the models. Non-regular fuzzy models can consists of any sector type, as of triangular-, tetragonal-, pentagonal-, hexagonal-sectors, etc. However, the tetragonal sector is the most important for non-regular modeling because it is basis for construction of higher-order sector models. Methods for construction of such models will be presented in next publications of the authors.

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