Robust exponential Stability in Mean Square for uncertain stochastic neural networks

Yiming Chen, Zenghui Xu, Zhiquan Zhou

(College of Science , Yanshan University , Qinhuangdao Hebei 066004 ,china,

E-mail：xuzenghui5566@163.com)

Abstract：A novel Lyapunov function is constructed to investigate the robust exponential stability in mean square for uncertain stochastic neural networks .A new criteria is derived in terms of linear matrix inequalities. The maximum value of the exponential convergence rate can be got from the criteria .The activation function is vary general , assuming neither differentiability nor strict monotonicity. The criteria can be checked easily by the LMI control toolbox in Matlab.A numerical example is given by LMI control toolbox in Matlab to demonstrate the effectiveness of our results.

Key words： Lyapunov stability theory ; Stochastic neural networks ; Robust exponential stability in mean square ;Linear matrix inequality (LMI)

1. Introduction

In many practical applications such as signal processing, optimization and control problems, the information to be processed is in the form of stable states. Therefore , in recent years, the study of stability analysis of neural networks has been attracting the interest of a great number of researchers [1-3] . On the other hand , it is known that time delays can not be avoided in the hardware implementation of neural networks. The existence of time delays may result in instability, oscillation and poor performances of neural networks. Therefore, the problem of stability of delayed neural networks have been extensively investigated [4-7] .

In the last few decades, it is often the case that the neural networks model possesses stochastic phenomenon. Therefore, there was a wide study on the stability analysis for delayed stochastic neural networks in recent years. So far, there are only a few papers that have taken stochastic phenomenon into account in neural networks [8-11].

Based on the above discussions, We consider a class of uncertain stochastic neural networks .The main purpose of this paper is to study the robust exponential stability in mean square. By using Lyapunov−Krasovskii functional we obtain the sufficient conditions for robust exponential stability in mean square of stochastic neural networks, in terms of linear matrix inequality (LMI). We also provide one example to demonstrate the effectiveness of the proposed stability results.

2. Problem statement

The model of uncertain stochastic neural networks can be expressed as follows:

$$dx(t)=[-A(t)x(t)+B_0(t)f(x(t))+B_1(t)f(x(t-\tau(t)))]dt+ \\ [C(t)x(t)+D_0(t)x(t-\tau(t))+D_1(t)f(x(t))+D_2(t)f(x(t-\tau(t)))]dw(t) \quad (1)$$

Where $x(t)=\left[x_1(t),x_2(t),\cdots x_n(t)\right]^T\in R^n$ is the state vector ,

$f(x(t))=[f_1(x_1(t)),f_2(x_2(t)),\cdots,f_n(x_n(t))]^T$ is the activation function;

$w(t)=[w_1(t),w_2(t),\cdots,w_m(t)]^T$ is m-dimensional Brownian motion defined on $(\Omega, F,\{F_t\}_{t\ge0},P)$.

Let $(\Omega, F,\{F_t\}_{t\ge0},P)$ be a complete probability space with a filtration $\{F_t\}_{t\ge0}$ satisfying the usual conditions .where $E(\cdot)$ stands for the mathematical expectation operator with respect to the given probability measure P. $E(dw(t))=0, E(dw^2(t))=dt$; $\tau(t)$ represents the time-varying delay of neural networks satisfying: $0\le\tau(t)\le\tau, \dot{\tau}(t)\le\mu<1$; $A(t)=A+\Delta A(t)$, $B_0(t)=B_0+\Delta B_0(t)$,

$B_1(t)=B_1+\Delta B_1(t)$, $C(t)=C+\Delta C(t)$, $D_0(t)=D_0+\Delta D_0(t)$, $D_1(t)=D_1+\Delta D_1(t)$

$D_2(t)=D_2+\Delta D_2(t)$; $A=diag(a_1,a_2,\cdots a_n)(a_i>0)$ B_0,B_1,C,D_0,D_1,D_2 are the interconnection matrices , $\Delta A(t),\Delta B_0(t),\Delta B_1(t),\Delta C(t),\Delta D_0(t),\Delta D_1(t),\Delta D_2(t)$ are the time-varying uncertainties of the form: $[\Delta A(t),\Delta B_0(t),\Delta B_1(t),\Delta C(t),\Delta D_0(t),\Delta D_1(t),\Delta D_2(t)]=MF(t)[N_1,N_2,N_3,N_4,N_5,N_6,N_7]$

Where M , N_i $(i=1,2,3,4,5,6,7)$ are known constant matrices of appropriate dimensions and $\mathbf{F}(t)$ is the time-varying uncertain matrices, which satisfies $\mathbf{F}^T(t)\ \mathbf{F}(t)\le\mathbf{I}$.

Note that the function $f_i(\cdot)(i=1,2,\cdots n)$ here are Bounded .It satisfies the following condition:

(A) $l_i^-\le\dfrac{f_i(y_1)-f_i(y_2)}{y_1-y_2}\le l_i^+$ for any l_i^- , $l_i^+(i=1,2,\cdots,n)$

Definition 1. The trivial solution of (1) is said to be globally robust exponential stability in mean square, if there exist constant $\eta>0$, $k>0$,such that: $E\|x(t)\|^2\le\eta e^{-2kt}\sup_{-\tau\le s\le0}E\|x(s)\|^2$

Lemma 1.For any constant symmetric positive-definite matrix M , a scalar $r>0$, the following inequality holds:

$$r\int_0^r w^T(s)Mw(s)ds\ge\left[\int_0^r w(s)ds\right]^T M\left[\int_0^r w(s)ds\right]$$

Lemma 2 .Let A,D,E,F and P be real matrices of appropriate dimensions , and $P>0$, $F^TF\le I$,

Then for any $\varepsilon>0$ making $p^{-1}-\varepsilon^{-1}DD^T>0$,the following inequality holds:

$$(A+DFE)^TP(A+DFE)\le A^T(P^{-1}-\varepsilon^{-1}DD^T)A+\varepsilon E^TE$$

Lemma 3 .Let U,V,W and $M=M^T$ be real matrices of appropriate dimensions , with V satisfying $V^TV\le I$,then $M+UVW+W^TV^TU^T<0$, If and only if there exists a positive scalar $\varepsilon>0$, such that $M+\varepsilon^{-1}UU^T+\varepsilon WW^T<0$

Lemma 4. (schur complement)For a given matrix $S=\begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$ with $S_{11}=S_{11}^T, S_{22}=S_{22}^T$,then the following conditions are equivalent:

1) $S<0$

2) $S_{22}<0, S_{11}-S_{12}S_{22}^{-1}S_{12}^T<0$

3) $S_{11}<0, S_{22}-S_{12}^TS_{11}^{-1}S_{12}<0$

3. Main Results and Proofs

Consider the following uncertain stochastic neural networks

$$\begin{aligned} dx(t) &= [-Ax(t)+B_0f(x(t))+B_1f(x(t-\tau(t)))]dt+ \\ &[\Delta C(t)x(t)+\Delta D_0(t)x(t-\tau(t))+\Delta D_1(t)f(x(t))+\Delta D_2(t)f(x(t-\tau(t)))]dw(t) \end{aligned} \quad (2)$$

In the following we denote $L_1=diag\left(l_1^-l_1^+, l_2^-l_2^+,\cdots,l_n^-l_n^+\right), L_2=diag\left(l_1^-+l_1^+, l_2^-+l_2^+,\cdots,l_n^-+l_n^+\right)$

Theorem 1. System (2) is robust exponential stability in mean square, if there exist positive matrices $P>0, Q_1>0, Q_2>0, R>0$, positive diagonal matrices $U_1=diag\left(u_{11},u_{12},\cdots u_{1n}\right)\ge 0$, $U_2=diag\left(u_{21},u_{22},\cdots u_{2n}\right)\ge 0$, and $\varepsilon_1>0$, such that

$$\Sigma_0=\begin{bmatrix} -P & PM \\ * & -\varepsilon_1 I \end{bmatrix}<0$$

$$\Sigma_1=\begin{bmatrix} \psi_{11} & \varepsilon_1N_4^TN_5 & PB_0+L_2U_1+\varepsilon_1N_4^TN_6 & PB_1+\varepsilon_1N_4^TN_7 \\ * & \psi_{22} & \varepsilon_1N_5^TN_6 & L_2U_2+\varepsilon_1N_5^TN_7 \\ * & * & -2U_1+R+\varepsilon_1N_6^TN_6 & \varepsilon_1N_6^TN_7 \\ * & * & * & -(1-\mu)e^{-2k\tau}R-2U_2+\varepsilon_1N_7^TN_7 \end{bmatrix}$$

$$\psi_{11}=2kP-PA-AP+Q_1+\tau^2Q_2-2L_1U_1+\varepsilon_1N_4^TN_4$$

$$\psi_{22}=-(1-\mu)e^{-2k\tau}Q_1-2L_1U_2+\varepsilon_1N_5^TN_5$$

Where * mean symmetric terms.

Proof For system(2),we choose the following Lyapunov– Krasovksii function

$$V(t)=e^{2kt}x^T(t)Px(t)+\int_{t-\tau(t)}^t e^{2ks}x^T(s)Q_1x(s)ds+\tau\int_{-\tau}^0\int_{t+\beta}^t e^{2ks}x^T(s)Q_2x(s)dsd\beta+\int_{t-\tau(t)}^t e^{2ks}f^T(x(s))Rf(x(s))ds$$

Where P , Q_1, Q_2 ,and R are positive matrices

By $It\hat{o}$ differential formula ,the stochastic derivation of $V(t)$ along（2）can be obtained as follows:

$$dV(t) \le e^{2kt}\{2kx^T(t)Px(t) + 2x^T(t)P\left[-Ax(t) + B_0 f(x(t)) + B_1 f(x(t-\tau(t)))\right] +$$
$$\left[\Delta C(t)x(t) + \Delta D_0(t)x(t-\tau(t)) + \Delta D_1(t) f(x(t)) + \Delta D_2(t) f(x(t-\tau(t)))\right]^T P \times$$
$$\left[\Delta C(t)x(t) + \Delta D_0(t)x(t-\tau(t)) + \Delta D_1(t) f(x(t)) + \Delta D_2(t) f(x(t-\tau(t)))\right] +$$
$$x^T(t)Q_1x(t) - (1-\mu)e^{-2k\tau}x^T(t-\tau(t))Q_1x(t-\tau(t)) + \tau^2 x^T(t)Q_2x(t) -$$
$$\tau e^{-2k\tau}\int_{t-\tau}^{t} x^T(s)Q_2x(s)ds + f^T(x(t))Rf(x(t)) - (1-\mu)e^{-2k\tau} f^T(x(t-\tau(t)))Rf(x(t-\tau(t)))\}dt +$$
$$\{2x^T(t)P\left[\Delta C(t)x(t) + \Delta D_0(t)x(t-\tau(t)) + \Delta D_1(t) f(x(t)) + \Delta D_2(t) f(x(t-\tau(t)))\right]\}dw(t)$$

From Lemma 1,we know that

$$-\tau\int_{t-\tau}^{t} x^T(s)Q_2x(s)ds \le -\tau\int_{t-\tau(t)}^{t} x^T(s)Q_2x(s)ds \le -\frac{\tau}{\tau(t)}(\int_{t-\tau(t)}^{t} x(s)ds)^T Q_2(\int_{t-\tau(t)}^{t} x(s)ds) \le -(\int_{t-\tau(t)}^{t} x(s)ds)^T Q_2(\int_{t-\tau(t)}^{t} x(s)ds)$$

From Lemma 2,we know that

$$\left[\Delta C(t)x(t) + \Delta D_0(t)x(t-\tau(t)) + \Delta D_1(t) f(x(t)) + \Delta D_2(t) f(x(t-\tau(t)))\right]^T P \times$$
$$\left[\Delta C(t)x(t) + \Delta D_0(t)x(t-\tau(t)) + \Delta D_1(t) f(x(t)) + \Delta D_2(t) f(x(t-\tau(t)))\right] =$$
$$\{MF(t)\left[N_4x(t) + N_5x(t-\tau(t)) + N_6 f(x(t)) + N_7 f(x(t-\tau(t)))\right]\}^T P \times$$
$$\{MF(t)\left[N_4x(t) + N_5x(t-\tau(t)) + N_6 f(x(t)) + N_7 f(x(t-\tau(t)))\right]\} \le$$
$$\varepsilon_1\left[N_4x(t) + N_5x(t-\tau(t)) + N_6 f(x(t)) + N_7 f(x(t-\tau(t)))\right]^T \times$$
$$\left[N_4x(t) + N_5x(t-\tau(t)) + N_6 f(x(t)) + N_7 f(x(t-\tau(t)))\right]$$

From Lemma 4,we know that

$P^{-1} - \varepsilon^{-1}MM^T > 0$ and $\begin{bmatrix} -P & PM \\ * & -\varepsilon_1 I \end{bmatrix} < 0$ are equivalent.

From（A）,we know that

$$\left[f_i(x_i(t) - l_i^- x_i(t)\right]\left[f_i(x_i(t) - l_i^+ x_i(t)\right] \le 0,\ f_i(0) = 0, i = 1,2,\cdots,n$$
$$\left[f_i(x_i(t-\tau(t)) - l_i^- x_i(t-\tau(t))\right]\left[f_i(x_i(t-\tau(t)) - l_i^+ x_i(t-\tau(t))\right] \le 0,\ f_i(0) = 0, i = 1,2,\cdots,n$$

there exist positive diagonal matrices, $U_1 = diag(u_{11}, u_{12}, \cdots, u_{1n}) \ge 0$ and $U_2 = diag(u_{21}, u_{22}, \cdots, u_{2n}) \ge 0$,such that

$$dv(t) \le dv(t) - 2\sum_{i=1}^{n} u_{1i}\left[f_i(x_i(t)) - l_i^- x_i(t)\right]\left[f_i(x_i(t)) - l_i^+ x_i(t)\right] -$$
$$2\sum_{i=1}^{n} u_{2i}\left[f_i(x_i(t-\tau(t))) - l_i^- x_i(t-\tau(t))\right]\left[f_i(x_i(t-\tau(t))) - l_i^+ x_i(t-\tau(t))\right] \le$$
$$\left\{\xi^T(t)\Sigma_1\xi(t)\right\}dt + \left\{2x^T(t)P\left[\Delta C(t)x(t) + \Delta D_0(t)x(t-\tau(t)) + \Delta D_1(t) f(x(t)) + \Delta D_2(t) f(x(t-\tau(t)))\right]\right\}dw(t)$$

Where $\xi(t) = \left[x^T(t), x^T(t-\tau(t)), f^T(x(t)), f^T(t-\tau(t))\right]$

From $\Sigma_1 < 0$ we can prove that a scalar $\gamma > 0$ satisfying $\Sigma_1 + diag(\gamma I, 0, 0, 0) < 0$ exists such that

$$\frac{dEV(t)}{dt} \le E\left[\xi^T(t)\Sigma_1\xi(t)\right] \le -\gamma E\|x(t)\|^2$$

So $EV(t) \le EV(0)$.

$$EV(0) = E\{x^T(0)Px(0) + \int_{-\tau(0)}^{0} e^{2ks} x^T(s)Q_1 x(s) + \tau\int_{-\tau}^{0}\int_{\beta}^{0} e^{2ks} x^T(s)Q_2 x(s)dsd\beta + \int_{-\tau(0)}^{0} e^{2ks} f^T(x(s))Rf(x(s))ds\} \le (\lambda_{\max}(P) + (\lambda_{\max}(Q_1) + \lambda_{\max}(R)L^T L)\int_{-\tau(0)}^{0} e^{2ks} x \, \tau\lambda_{\max}(Q_2)\int_{-\tau}^{0}\int_{\beta}^{0} e^{2ks} dsd\beta) \sup_{-\tau\le s\le 0} E\|x(s)\|^2$$

$$L = diag(\max(|l_i^-|, |l_i^+|), i = 1, 2, \cdots, n$$

Also, we have $EV(t) \ge \lambda_{\min}(P) e^{2kt} E\|x(t)\|^2$

Therefore, we have $E\|x(t)\|^2 \le \eta e^{-2kt} \sup_{-\tau\le s\le 0} E\|x(s)\|^2$

We can come to a conclusion that (2) is robust exponential stability in mean square.

Next, we consider the following uncertain stochastic neural networks

$$\begin{aligned} dx(t) &= [-Ax(t) + B_0 f(x(t)) + B_1 f(x(t-\tau(t)))]dt + \\ &[C(t)x(t) + D_0(t)x(t-\tau(t)) + D_1(t) f(x(t)) + D_2(t) f(x(t-\tau(t)))]dw(t) \end{aligned} \quad (3)$$

Theorem 2. System (3) is robust exponential stability in mean square, if there exist positive matrices $P > 0, Q_1 > 0, Q_2 > 0, R > 0$, positive diagonal matrices $U_1 = diag(u_{11}, u_{12}, \cdots u_{1n}) \ge 0$,

$U_2 = diag(u_{21}, u_{22}, \cdots u_{2n}) \ge 0$, and $\varepsilon_1 > 0$, such that

$$\Sigma_0 = \begin{bmatrix} -P & PM \\ * & -\varepsilon_1 I \end{bmatrix} < 0$$

$$\Sigma_2 = \begin{bmatrix} \phi_{11} & \varepsilon_1 N_4^T N_5 + C^T PD_0 & PB_0 + L_2U_1 + \varepsilon_1 N_4^T N_6 + C^T PD_1 & PB_1 + \varepsilon_1 N_4^T N_7 + C^T PD_2 \\ * & \phi_{22} & \varepsilon_1 N_5^T N_6 + D_0^T PD_1 & L_2U_2 + \varepsilon_1 N_5^T N_7 + D_0^T PD_2 \\ * & * & -2U_1 + R + \varepsilon_1 N_6^T N_6 + D_1^T PD_1 & \varepsilon_1 N_6^T N_7 + D_1^T PD_2 \\ * & * & * & -(1-\mu)e^{-2k\tau} R - 2U_2 + \varepsilon_1 N_7^T N_7 + D_2^T PD_2 \end{bmatrix}$$

$$\phi_{11} = 2kP - PA - AP + Q_1 + \tau^2 Q_2 - 2L_1U_1 + \varepsilon_1 N_4^T N_4 + C^T PC$$

$$\phi_{22} = -(1-\mu)e^{-2k\tau} Q_1 - 2L_1U_2 + \varepsilon_1 N_5^T N_5 + D_0^T PD_0$$

Where * mean symmetric terms.

Proof. The Lyapunov–Krasovskii functional is the same as Theorem 1.

Last, we consider the following uncertain stochastic neural networks

$$\begin{aligned} dx(t) &= [-A(t)x(t) + B_0(t) f(x(t)) + B_1(t) f(x(t-\tau(t)))]dt + \\ &[C(t)x(t) + D_0(t)x(t-\tau(t)) + D_1(t) f(x(t)) + D_2(t) f(x(t-\tau(t)))]dw(t) \end{aligned} \quad (3)$$

Theorem3. System (3) is robust exponential stability in mean square, if there exist positive matrices $P > 0, Q_1 > 0, Q_2 > 0, R > 0$, positive diagonal matrices $U_1 = diag(u_{11}, u_{12}, \cdots u_{1n}) \ge 0$,

$U_2 = diag(u_{21}, u_{22}, \cdots u_{2n}) \ge 0$, and $\varepsilon_1 > 0$, $\varepsilon_2 > 0$, such that

$$\Sigma_0 = \begin{bmatrix} -P & PM \\ * & -\varepsilon_1 I \end{bmatrix} < 0$$

$$\Sigma_3 = \begin{bmatrix} \varphi_{11} & \varepsilon_1 N_4^T N_5 + C^T PD_0 & PB_0 + L_2 U_1 + \varepsilon_1 N_4^T N_6 + C^T PD_1 - \varepsilon_2 N_1^T N_2 & \varphi_{14} & PM \\ * & \varphi_{22} & \varepsilon_1 N_5^T N_6 + D_0^T PD_1 & L_2 U_2 + \varepsilon_1 N_5^T N_7 + D_0^T PD_2 & 0 \\ * & * & -2U_1 + R + \varepsilon_1 N_6^T N_6 + \varepsilon_2 N_2^T N_2 & \varepsilon_1 N_6^T N_7 + D_1^T PD_2 + \varepsilon_2 N_3^T N_3 & 0 \\ * & * & * & \varphi_{44} & 0 \\ * & * & * & * & -\varepsilon_2 I \end{bmatrix}$$

$$\phi_{11} = 2kP - PA - AP + Q_1 + \tau^2 Q_2 - 2L_1 U_1 + \varepsilon_1 N_4^T N_4 + C^T PC + \varepsilon_2 N_1^T N_1$$

$$\phi_{22} = -(1-\mu)e^{-2k\tau} Q_1 - 2L_1 U_2 + \varepsilon_1 N_5^T N_5 + D_0^T PD_0$$

$$\varphi_{14} = PB_1 + \varepsilon_1 N_4^T N_7 + C^T PD_2 - \varepsilon_2 N_1^T N_3$$

$$\varphi_{44} = -(1-\mu)e^{-2k\tau} R - 2U_2 + \varepsilon_1 N_7^T N_7 + D_2^T PD_2 + \varepsilon_2 N_3^T N_3$$

Where $*$ mean symmetric terms.

Proof. The Lyapunov–Krasovskii functional in the proof of Theorem 1 is used, and A, B_0, B_1 are replaced by $A + \Delta A(t)$, $B_0 + \Delta B_0(t)$ and $B_1 + \Delta B_1(t)$, respectively.

We can known that

$$\Sigma_4 = \Sigma_2 + \begin{bmatrix} M^T P & 0 & 0 & 0 \end{bmatrix}^T F(t) \begin{bmatrix} -N_1 & 0 & N_2 & N_3 \end{bmatrix} + \begin{bmatrix} -N_1 & 0 & N_2 & N_3 \end{bmatrix}^T F(t) \begin{bmatrix} M^T P & 0 & 0 & 0 \end{bmatrix} < 0$$

From Lemma 3,we know that $\Sigma_4 < 0$ are equivalent to

$$\Sigma_5 = \Sigma_2 + \varepsilon_2^{-1} \begin{bmatrix} M^T P & 0 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} M^T P & 0 & 0 & 0 \end{bmatrix} + \varepsilon_2 \begin{bmatrix} -N_1 & 0 & N_2 & N_3 \end{bmatrix}^T \begin{bmatrix} -N_1 & 0 & N_2 & N_3 \end{bmatrix} < 0$$

From Lemma 4,we know that Σ_5 and Σ_3 are equivalent

So, the uncertain stochastic neural networks (3) is robust exponential stability in mean square

4. Example

In this section, one example is given to show the effectiveness of our theoretical results.

Consider the uncertain stochastic neural networks

$$\begin{aligned} dx(t) &= [-A(t)x(t) + B_0(t) f(x(t)) + B_1(t) f(x(t-\tau(t)))]dt + \\ &[C(t)x(t) + D_0(t)x(t-\tau(t)) + D_1(t) f(x(t)) + D_2(t) f(x(t-\tau(t)))]dw(t) \end{aligned}$$ with the following parameters:

$$A = \begin{bmatrix} 2.2 & 0 & 0 \\ 0 & 2.4 & 0 \\ 0 & 0 & 2.6 \end{bmatrix}, B_0 = 0, B_1 = \begin{bmatrix} 0.5 & 0.6 & 0.9 \\ 1.7 & 1.9 & 1.8 \\ 1.3 & 1.5 & 1.9 \end{bmatrix}, M = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.3 \end{bmatrix}, C = D_0 = D_1 = D_2 = 0$$

$N_1 = 0.6 I_3$, $N_2 = 0$, $N_3 = 0.2 I_3$, $N_4 = 0.2 I_3$, $N_5 = 0.2 I_3$, $N_6 = N_7 = 0$, $\sigma = 0.8$, $l_i^- = 0$,

$l_i^+ = 0.5, i = 1, 2, 3$

$\mu = 0$, By solving the LMIS in Σ_0 and Σ_3 , it can be proved that the uncertain stochastic neural networks is robust exponential stability in mean square .The maximum value of the exponential convergence rate can be got. The maximum value of the exponential convergence rate $k = 0.0670$ 。A set of feasible solution are as follows：

$$P = \begin{bmatrix} 2313.8 & 525.3 & -588.4 \\ 525.3 & 696.2 & 191.1 \\ -588.4 & 191.1 & 143.5 \end{bmatrix}, \quad Q_1 = \begin{bmatrix} 3747.1 & 649.7 & -1203.8 \\ 649.7 & 948.7 & 42.7 \\ -1203.8 & 42.7 & 2387.9 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 2274.9 & 386 & -1623.1 \\ 386 & 102 & -324.3 \\ -1623.1 & -324.3 & 1223.8 \end{bmatrix}, \quad R = \begin{bmatrix} 3924.1 & 1362.3 & 346.5 \\ 1362.3 & 2161.1 & 1286 \\ 346.5 & 1286 & 3760.9 \end{bmatrix},$$

$$U_1 = \begin{bmatrix} 8373.5 & 0 & 0 \\ 0 & 3612.9 & 0 \\ 0 & 0 & 5652.3 \end{bmatrix}, U_2 = \begin{bmatrix} 10923 & 0 & 0 \\ 0 & 4308 & 0 \\ 0 & 0 & 6622 \end{bmatrix},$$

$\varepsilon_1 = 949.1611$, $\varepsilon_2 = 190.0355$

5. Conclusion

In this paper, a new criterion is derived by LMIS to ensure the robust exponential stability in mean square for the uncertain stochastic neural networks. The activation function is vary general ,assuming neither differentiability nor strict monotonicity . What is more ,we can get the maximum value of the exponential convergence rate from it.The effectiveness of the proposed criterion is demonstrated in numerical example.

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