ON THE STUDY OF GAME THEORY

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ABSTRACT

This paper deals with Game Theory. The fascination of the game theory emerges from the fact that it shows us how we cannot simply derive conclusions about outcomes in competitive settings from psychological facts about the competitors. The complete set utility function, along with specifications about the extent to which the agents are privy to one and there are utility functions, determines the equilibrium strategies available to them. These and many more are studies in this work.

KEYWORDS: Game theory, Utility functions, Equilibrium Strategies, Payoff Function, Players, Specifications, Agents.

INTRODUCTION

The Origin of games has been vaguely assigned to the inborn tendency of mankind to amuse itself. Games have no geometrical boundaries and game playing ie found in all parts of the world whether it be in the under developed areas of Africa or in developed countries (Adeosun and Adetunde (2008)).

The Babylonian Tumud is the compilation of ancient law and tradition set down during the first five centuries A.D. which serves as the basis of Jewish religious, criminal and civil law. In 1985, it was recognized that the Talmud anticipates the modern theory of cooperative games. Each solution corresponds to the nucleolus of an appropriately defined game.

In a letter dated November 1713 James Waldegrave provided the first, known, minmax mixed strategy solution to a two-person game. Waldegrave wrote the letter, about a two-person version of the card game le Her, to Pierre-Remond de Montmort who in turn wrote to Nicolas Bernoulli, incuding in his letter a discussion of the Waldegrave solution. Waldegrave's solution is minimax mixed strategy equilibrium, but he made no extension of his result to other games, and expressed concern that a mixed strategy "does not seem to be in the usual rules of play" of games of chance.

The first theorem of game theory asserts that chess is strictly determined, i.e chess has only one individually rational payoff profile in pure strategies. This theorem was published by Zermelo (1913) in his paper Uber eine Anwendung der Mengenlehre auf die Theorie des schachspiels and hence is referred to as Zermelo's Theorem.

Emile Borel published four notes on strategies of games and an erratum to one of them. Borel gave the first modern formulation a mixed strategy along with finding the minimax solution for two-person games with three or five possible strategies. Initially he maintained that games with more possible strategies would not have minimax solution, but by 1927, he considered this an open question as he had been unable to find a counter example.

John Von Neumann (1928) proved the minimax theorem in his article Zur Theorie der Gessellschaftsspiele. It states that every two-person zero-sum game with finitely many pure strategies for each player is determined, i.e when mixed strategies are admitted, this variety of game has precisely one individually rational payoff vector. The proof makes use of some topology and of functional calculus. This paper also introduced the extensive form of game.

Publication of F. Zenthen's book (1930) "Problems of Monopoly and Economic Warfare". In Chapter IV he proposed a solution to the bargaining solution to the bargaining problem which Harsanyi later showed is equivalent to Nash's bargaining solution.

Fisher, R.A (1934) independently discovers Waldegrave's solution to the card game le Her. Fisher reported his work in the paper "Randomisation and an Old Enigma of card Play"

John Von Neumann and Oskar Morgentem publication (1944) expound two-person zero-sum theory, this book is the senminal work in area game theory such as the notion of a cooperative game, with transferable utility (TU), its coalitional form and its Von Neumann-Morgenstem stable sets. It was also the account of axiomatic utility theory given here thet led to its wide spread adoption within economics.

In January 1950 Melvin Dresher and Merrill Flood carried out, at the Rand Corporation, the experiment which introduced the game now known as the Prisoner's Dilemma. The famous story associated with this game is due to A.W. Tucker, 'A Two-person Dilemma', (memo, Standford University). Howard Raiffa independently conducted, unpublished, experiments with the Prisoner's Dilemma.

In four papers between 1950 and 1953 John Nash made seminar contributions to both non-cooperative game theory and to bargaining theory. In two papers, "Equilibrium Points in N-Person Games (1950) and Non-cooperative Games (1951)", Nash prove the existence of a strategic equilibrium for non-cooperative games – the Nash equilibrium – and proposed the "Nash program", in which he suggested approaching the study of cooperative games via their reduction to non-cooperative form. In his two papers on bargaining theory, "The Bargaining problem" (1950) and "Two-person Cooperative Games" (1953), he founded axiomatic bargaining theory, proved the existence of the Nash bargaining solution and provided the first execution of the Nash program.

George W. Brown (1951) described and discussed a simple iterative method for approximating solutions of discrete zero-sum games in his paper "Iterative Solutions of Games by Fictitious Play."

John Charles C. Mckinscy (1952) published the first textbook on game theory "Introduction to the Theory of Games".

The notion of the Core as a general concept was developed by Shapley, L.S and Gillies, D.B (Some Theorems on N-Person Games). This Core is the set of allocations that cannot be improved upon by any coalition.

Lloyd Shapley (1953) in his paper "A value for N-Person games" characterized, by a set of axioms, a solution concept that associates with each coalitional game, v, a unique out-come, v. This solution is now known as the Sharpley value.

Also, Lloyd Shapley (1953) in his paper "Stochastic games" showed that for the strictly competitive case, with future payoff discounted at a fixed rate, such games are determined and that they have optimal strategies that depend only on the game being played, not on the history or even on the date, i.e. the strategies are stationary.

Extensive form games allow the modeler to specify the exact order in which players have to make their decisions and to formulate the assumptions about the information possessed by the players in all stages of the game Kuhn's H.W (1953) paper "Extensive Games and the problem of Information" includes the formulation of extensive form games which is currently used, and also some basic theorems pertaining to this class of games.

Differential Games were developed by Rufus Isaacs in the early 1950s. they grew out of the problem of forming and solving military pursuit games.

The notion of a strong Eqilibrium was introduced by R.J. Aumann (1959) in the paper "Acceptable Points in General Cooperative N-Person Games".

The relationship between Edgeworth's idea of the contract curve and the Core was pointed out by Martin Shubik (1959) in his paper "Edgeworth Market games". One limitation with this paper is that Shubik worked within the confines of TU games whereas Edgeworth's idea is more appropriately modeled as an NTU game.

One of the first books to take an explicitly non-cooperative game theoretic approach to modeling oligopoly is the publication of Martin Shubik's "Strategy and Market Structure: Competition, Oligopoly, and the Theory of Games". It also contains an early statement of the Folk Theorem.

Near the end of this decade came the first studies of repeated games. The main result to appear at this time was the Folk Theorem. This states that the equilibrium outcomes in an infinitely repeated game coincide with the feasible and strongly individually rational outcomes of the one-shot game on which it is based.

The development of NTU (non-transferable utility) games made cooperative game theory more widely applicable. Von Neumann and Morgenstern stable sets were investigated in the NTU context in the Aumann and Peleg (1960) paper "Von Neumann and Morgenstern solutions to cooperative Games without side Payments".

In Kari Borch (1962) paper "Automobile Insurance", the article indicates how game theory can be applied to determine premiums for different classes of insurance, when required total premiums for all classes are given. Borch suggests that the Shapley value will give reasonable premiums for all classes of risk.

Bondareva, O.N. (1963) established that for a TU game its core is non-empty if and only if it is balanced. The reference, translates as some Applications of Linear Programming Methods to the Theory of Cooperative Games.

Aumann, R.J (1964) introduced and discussed idea of the Bargaining Set in his paper "The Bargaining Set for Cooperative Games". The bargaining set includes the core but unlike it, is never empty for TU games.

Carlton E. Lemke and J.T. Howson, Jr (1964) describe an algorithm for finding a Nash equilibrium in a bimatrix game. Thereby giving a constructive proof of the existence of an equilibrium point, in their paper "Equilibrium Points in Bimatrix Games". The paper also shows that, except for degenerate situations, the number of equilibra in a bimatrix game is odd.

Selten, R (1965) in his article "Spielheoretische Behandlung eines Oligopolmodellsmit Nachfragetraegheit" introduced the idea of refinements of the Nash equilibrium with the concept of (Subgame) perfect equilibra.

Infinitely repeated game with incomplete information were born in a paper by Aumann, R.J. and Maschler, M. in 1966 titled "Game-Theoretic Aspects of Gradual Disarmament".

In his paper "A General Theory of Rational Behaviour in Game Situations". John Harsanyi (1966) gave the, now, most commonly used definition to distinguish between cooperative and non-cooperative games. A game is cooperative if commitments – agreements, promises, threats – are fully binding and enforceable. It is non-cooperative if commitments are not enforceable.

In the article “The Core of a N-Person Game”, Scarf, H.E. (1967) extended the notion of balancedness to NTU games, then showed that every balanced NTU game has a non-empty core.

In a series of three papers, “Games with Incomplete Information Played by Bayesian Players” Part I, II and III, John Harsanyi constructed the theory of games of incomplete information. This laid the theoretical groundwork for information economics that has become one of the major themes of economics and game theory.

William Lucas (1968) in his paper “A Game with no Solution” answered the long-standing question as to whether stable sets always exist.

David Schmeidler (1969) introduced the Nucleolus in his paper “The Nucleolus of characteristic Game”. The Nucleolus always exits, is unque, is a member of the Kermel and for any non-empty core is always in it.

For a coalitional game to be a market game it is necessary that it and all its subgames must have non-empty cores, i.e. that the game be totally balanced. In “Market Games” L.S. Shapley and Martin Shubik (1969) prove that this necessary condition is also sufficient.

In 1972, Oskar Morgenstern founded International Journal of Game Theory. John Maynard Smith (1972) introduced the concept of an Evolutionarily stable strategy (ESS) to evolutionary game theory in an essay ‘Game theory and the Evolution of Fighting’. The ESS concept has since found increasing use within the economics (and biology) literature.

In the traditional view of strategy randomization, the players use a randomizing device to decide to their actions. John Harsanyi (1973) was the first to break away from this view with his paper “Games with Randomly Disturbed Payoffs: A New Rationale for Mixed Strategy Equilibrium Point”. For Harsanyi, nobody really randomizes. The appearance of randomization is due to the payoff being exactly known to all; each player, who known his own payoff exactly, has a unque optimal action against his estimate of what the others will do.

Publication of Aumann R.J and Shapley L.S (1974) “values of Non-Atomic Games” deals with values for large games in which all the players are individually insignificant (non-atomic games). Aumann R.J. (1974) proposed the concept of a correlated equilibrium in his paper “Subjectivity and Correlation in Randomized Strategies”.

The introduction of trembling hard perfect equilibria occurred in the paper “Reexamination of the Perfectness Concept for Equilibrium Points in Extensive games by Reinhard Selten (1975)”. This paper was the true catalyst for the “refinement industry” that has developed around the Nash equilibrium.

Kalai E. and Smorodinsky M. (1975), in their article “Other Solutions to Nash’s Bargaining Problem”, replace Nash’s independence of irrelevant alternatives axiom with a monotonicity axiom. The resulting solution is known as the kalai-Smorodinsky solution.

Littlechild S.C and Thompson G.F (1977) are among the first to apply the nucleous to the problem of cost allocation with article “Aircraft Landing Fees: A game Theory Approach”. They use the nucleolus, along with the Core and Shapley value, to calculate fair and efficient landing and take-off fees for Birmingham airport.

Aumann, R.J. (1981) published a survey of Repeated Games. This survey firstly proposed the idea of applying the notion of an automaton to describe a player in a repeated game. A second idea from the survey is to study the interactive behaviour of bounded players by studying a game with appropriately restricted set of strategies. These ideas have given birth to a large and growing literature.

Divid M. Kreps and Robert Wilson (1982) extend the idea of a subgame perfect information. They call this extended idea of equilibrium sequential. It is detailed in their paper “Sequential equilibria”.

Rubinstein, A (1982) considered a non-cooperative approach to bargaining in his paper “Perfect Equilibrium in a Bargaining Model”. He considered an alternating offer game where offers are made sequentially until one is accepted. There is no bound on the number of offers that can be made but there is a cost to delay for each player. Rubinstein showed that the subgame perfect equilibrium is unique when each player’s cost of time is given by some discount factor delta.

Following the work of Gale and Shapley, A.E. Roth (1984) applied to hospitals. In his paper “The Evolution of the Labour Market for Medical Interns and Residents: A case Study in Game Theory” he found that American hospitals developed in 1950 a method of assignment that is a point in the core.

For a Bayesian game the question arises as to whether or not it is possible to construct a situation for which there is no sets of types large enough to contain all the private information that players are supposed to have. In their paper “formulation of Bayesian Analysis for games with Incomplete Information” J.F Mertens and Zamir, S (1985) show that it is not possible to do so.

Following Aumann, the theory of automata is now being used to formulate the idea of bounded rationality in repeated games. Two of the first articles to take this approach were A. Neyman’s 1985 paper “Bounded Complexity Justifies Cooperation in the Finitely Repeated Prisoner’s Dilemma”

A few games that have been programmed for play on digital computers are identified below. There are rules for playing these games:

1) Tic-Tac-Toe

Many special purpose machines of today now play Tic-Tac-Toe game. TheTic-Tac-Toe program have been written for many digital computers. CharlesBabbage conceived as far back as 1800's the idea of playing Tic-Tac-Toe on amachine.

2) Go

This Japanese game is a very popular game among computer people. The game isPlayed with black and white stones on a board containing 361 intersection points. The rules of Go are simple and no mathematical theory of the game is known. It is Estimated that there are around 10^{172} different board positions during the course of a game. It is easily seen that it would be impossible to calculate all the various borad configurations during the course of a game. This is one of the reasons that GO is such an interesting game to play on a computer.

3) Pentominoes

A polyomino is a figure formed by joining unit squares along their edges. Pentominoes are five-square polyominoes and it is possible to construct 12 different pentominoes. A pentomino game is played by arranging the 12 pentomines into various size rectangular boxes: 3 by 20, 4 by 15, 5 by 12, or 6 by 10. Computers have been used to generate many solution to the pentomio game. In fact, a computer program found that there are two solutions for the 3 by 20 configuration, 1010 for the 5 by 12 configuration and 2339 for the most popular size, the 6 by 10 rectangular configuration.

4) Knight's Tour

The strange moves of the Chess Knight make his operations fascinating. We are permitted to move two or one rows up or down and one or two columns left or right on the Chessboard. An interesting game is to move the knight to every square on the chessboard without landing in any square twice. There are many different tours and digital computers have been used to determine many of them.

5) Go-Moko

Go-Moko is a two-player game played on a by 19 lined Go board. Each player has 180 stones and places the stones, on alternate moves on an intersection of the board. The object is to obtain five adjacent stone in a row either vertically, horizontally, or diagonally. The player doing this wins the game. Several computer programs have been written to play this game.

6) Puzzle

It consists of a square box containing squares with the numbers 1 to 15 and one blank square. Any one of the numbers to the immediate right, left, top, or bottom of the blank square can be moved into the blank space. The object of the puzzle is to start with a specific number arrangement and finish with a different arrangement. There is one slight catch to the Puzzle there are 10, 461, 394, 944, 000 number arrangements that are impossible to obtain. There are also the same numbers of possible arrangements. A computer program of around 100 machine language statements determine if a specified number arrangement of the 15 puzzle is possible or impossible.

7) NIM

This is an ancient mathematical game. It is played by two people or one person and computer playing alternately. Before the play starts, an arbitrary number of objects is put in an arbitrary number of piles, in no specific order. Then each player in his turn removes as many objects as he wishes from any pile (but from only one pile) and at least one object. The player who takes the test chip is the winner of the game.

8) Slot Machines

A computer is used to simulate the operation of a slot machine. Instead of pulling the handle as one would do on a real Slot Machine, the action was started by pointing alight pen at a start position on the display console. The computer generated a three-symbol combination composed of the following symbols: chaerries, oranges, melons, bars, bells, lemons and plums. This symbol combination, along with an indicated payoff, was displayed on the cathode ray tube of the display console.

The computer system provides a printed listing of the Slot's identification, the money invested in the machine by slot enthusiasts and the amount of payoff. The computer can easily keep track of the operation several hundred Slot Machines.

9) Prime Numbers

An integer greater than one is called a Prime Number if and only if the only positive integers that exactly divide it are itself and the number one. How does one determine if anumber is prime? One way is to write down a large number of integers and simply cross off the composite numbers (numbners that are divisible by numbers other than themselves and the number 1). This simple procedure is relatively easy to use when one wants to determine only a few Prime Numbers; however, it would be a rather lengthy operation to determine all the prime less than 200,000 or to determine if 209267 is a prime number. A computer can easily determine if a number is prime by using a method similar to that of Eratostheness. A computer was used to determine a 961 – digit prime number (2^{11213} - 1) was a 3376 – digit Prime number.

10) Magic Squares

Magic squares were known to the ancients and were thought to possess mystic and magical powers because of their unsual nature. These magical squares have little practical value; however, they provide stimulating problems for programmer training. Other games that have been programmed for play on digital computers are: War Games, Checkers, Chess, Blackjack, Roulette, etc.

2. TYPES OF GAMES

There are various types of gaming activities. The simplest type of game is one which has only two players, and where the gain of one is the loss of the other. Such a game is called a zero-sum two-person game.

2.1 Two-Person Game

A game that involves only two players is called a two-person game. A player cannot play it and the number of players must not exceed two that is, two player are required to play this type of game at a time.

2.1.1 Zero-Sum Game

This is the type of game whereby the sum of the gains together with the losses equal to zero. Here, the gains (payoffs) equal the losses (payoffs). If the gains are represented as positive values, the loss will be represented as negative values; they both have the same magnitude.

2.1.2 Zero-Person Two-Person Game

This is a game involving only two players. In this type of game, there is just a play and the game is over. A player will lose and the other will gain if both use their best strategies thus, resulting in zero-sum when the payoffs to both are added together.

2.1.3 N-Person Game

This is a game involving more than two players. This type of game does not give a zero-sum game that is, the magnitude of gains or losses to each player is not equal. In fact, individual player is rated according to his or her performance and at the end the results are computed. The chance of playing the game is more than one before results are computed as against the two-person game.

2.1.4 Non Zero-Sum Game

Any game that has its result not equal to zero is called non zero-sum game. The payoffs of the players when added together give no zero-sum. That is, game and losses when added together give no zero-sum result.

2.2 COMPETITIVE SITUATION

A competitive situation is called a game if it has, for example, the following properties:

a) There are a finite number of participants, called players.
b) Each player has a finite number of possible courses of action.
c) A play occurs when each player chooses one of his courses of action (The choices are assumed to be made simultaneously, i.e., no player knows the choice of another until he has decided on his own).

d) Every combination of courses of action determines an outcome which results in a gain to each player. (A loss is considered a negative gain).

2.3 SOLUTION OF A GAME

The solution of a game involves finding:

a) The best strategies for both players.
b) The value of the game.

In this situation, both players use their best strategies that are stable in the sense that neither player can increase his gain by deviating from his initial strategy once he becomes aware of his opponent's.

2.4 STRATEGY OF A PLAYER

The strategy of a player is the decision rule he uses to decide which course of action he should employ. This strategy may be pure strategy or mixed strategy.

2.4.1 Pure Strategy

A pure strategy is a decision always to select the same course of action.

2.4.2 **Mixed Strategy**

A mixed strategy is a decision to choose at least two of his courses of action with fixed probabilities, i.e. if a player decides to use just two courses of action with equal probability, he might spin a coin to decide which one to choose. The advantage of a mixed strategy is that an opponent is always kept guessing as to which course of action is to be selected on any particular occasion.

2.5 BEST STRATEGY

We define "best strategy" on the basis of the minimax criterion of optimality explained below. This states that if a player lists all his possible payoffs of all his potential courses of action he will choose that course of action which corresponds to the best of his outcome. The implication of this criterion is that the opponent is an extremely shrewd player who will ensure that, whatever any course of action picked, our gain is kept to a minimum.

2.6 VALUE OF A GAME

The value of a game is the expected gain of player A if both players use their best strategies.

2.6.1 **Minimax Criterion of Optimality**

Best strategy is defined on the basis of the minimax criterion of optimality. This states that if a player lists the worst possible outcomes of all his potential strategies, he will choose that strategy which corresponds to the best of these worst outcomes. The implication of this criterion is that the opponent is an extremely shrewd player who will ensure that, whatever our strategy, our gain is kept to a minimum.

2.7 STABLE SOLUTION

A stable solution can only exit in terms of pure strategies when the payoff matrix has a saddle point.

If there is no such saddle point the strategies are mixed strategies and the problem becomes one of evaluating the probabilities with which each course of action should be selected.

Consider the following game of matching Pennies. Two players, A and B each put down a penny. If the coins match, i.e. both are heads or both are tail. A collects them both; otherwise B collects them both. The payoff matrix for this game is given below:

$$\begin{array}{ccc} & & \textit{Player B} \\ & & \begin{array}{cc} I & II \\ (Heads) & (Tails) \end{array} \\ \textit{Player A} & \begin{array}{c} I\ (Heads) \\ II\ (Tails) \end{array} & \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \end{array}$$

Intuitively, it can be seen that it is not a good plan for either player to decide in advance to play either of his pure strategies. Success in this game lies in attempting to anticipate the opposing player's course of action. A player could score over his opponent if he detected any pattern in his opponent's strategy or noticed that his opponent had a preference for either heads or tails. The opponent may only obviate such detection by selecting his courses of action at random such that the probability of choosing either heads or tails is $\frac{1}{2}$. Such a strategy may be represented as $\left(\frac{1}{2},\frac{1}{2}\right)$. A player may employ this strategy, for example by tossing the coin. If player A used his strategy he would win, on average, as often as he would lose, and his average or expected gain would be zero. This would be true whatever strategy player *B* adopted, whether he played heads throughout, tails throughout, or used the same strategy as *A*. if player *A* uses the strategy $\left(\frac{1}{2},\frac{1}{2}\right)$ he cannot lose whatever *B* decide to do. Similar reasoning also hold for player B. as there is no strategy for either player which will ensure a positive gain, the strategy $\left(\frac{1}{2},\frac{1}{2}\right)$ is the optimal strategy for both players according to the minimax criterion. The situation where both players use this strategy is stable in the sense that when either player realizes the other's strategy he has no incentive to change his own. This intuitive analysis affords s clue to the solution of games which do not have saddle points.

2.8 WEIGHTED AVERAGE OF THE POSSIBLE OUTCOMES

Consider the game with the following payoff matrix

$$\begin{array}{cc} & \textit{Player B} \\ & \begin{array}{cc} I & II \end{array} \\ \textit{Player A} \begin{array}{c} I \\ II \end{array} & \begin{bmatrix} a & b \\ c & d \end{bmatrix} \end{array}$$

If this game is to have no saddle point the two largest elements of the matrix must constitute one of the diagonals. We assume that this is so and, therefore both players use mixed strategies. Our task is to determine the probabilities with which both players choose their course of action. Let player *A* use his first course of action with probability x and, therefore his second course of action with probability *(1-x)*. Let player *B's* strategy, similarly, be *(y, 1-y)*. The expected gain to *A* if *B* plays his course of action *I* throughout is *ax + c(1-x)*. Similarly, the expected gain to *A* if *B* plays his course of action *II* throughout is *bx + d(1-x)*. Thus *A's* expected gain if *B* plays *(y, 1-y)* is

$$y[ax + c(1-x)] + (1-y)[bx + d(1-x)]$$

2.9 **Informal Definition**

A game is a set of acts by 1 to n rational Dennettian agents and possibly an a rational Dennettian agent (a random mechanism) called nature where at least one Dennettian agent has control over the outcome of the set of acts and where the Dennettian agents are potentially in conflict, in the sense that one Dennettian agent could rank outcomes differently from the others. A strategy for a particular Dennettian agent *i* is a vector that specifies the acts that *i* will take in response to all possible act by other agents. A Dennettian agent *i* is rational if and only if for given strategies of other agents the set of acts specified by *i's* strategy is such that it secures the available consequence which is most highly ranked by *i*. Nature is a generator of probabilistic influences on outcomes: technically it is the unique Dennettian agent in a game that is not rational.

Dennettian Agents: A Dennettian agent is a unit acts, where an act is any move that potentially influences future allocations.

Game may be represented either ion extensive form, that is, using a "tree" structure of the sort that is familiar to decision theorists, where each player's strategy is a path through the tree, or in strategic form. A game in strategic form is a list: $G = \{N, S, (s)\}$

Where

- ***N*** is the set of players and the index I designates a particular agent i. $N = \{0,1,2,3,....,n\}$
- ***S*** is the strategy space for the agents $S = Xni = OSi$

  Where

  Si is the set of possible strategies for *i*.
- **(*s*)** is a vector of payoff function one for each agent, excluding player O. each payoff function specifies the consequence for the agent in question of the strategies specified for all agents. $(s) = (1(s),....,n(s))$

2.10 APPLICATIONS

Game theory has of course, been extensively used in microeconomic analysis where its record of accurate predictions has been impressive in areas such as industrial organization theory, the theory of the firm, and auction theory. In macroeconomics and political science its use has been more controversial, since in such applications it is often difficult to establish that the specified game is in fact an accurate representation of the empirical phenomenon being modeled. For example, it has been common place to suggest that the nuclear standoff between the United States and the Soviet Union during the cold war was a case of the Prisoner's Dilemma. However it is far from obvious that the leaderships in either country in fact attached the necessary payoffs in their utility functions – preferring the destination of the world to their own unique destination. Game Theory has also been fruitfully applied in evolutionary biology, where species and/or genes are treated as players.

3.0 SOLUTION STRATEGIES

3.1 PAYOFF DETERMINATION/CALCULATION

One play of the game consists of a simultaneous selection of one A_i by player A and one B_i by player B. thus is the end of the game and the payoff is then determined

$$Player\ A's\ \ Payoff\ = \alpha_{ij}$$
$$Player\ B's\ \ Payoff\ = -\alpha_{ij}$$
$$Or$$
$$Player\ A's\ Payoff\ = -\alpha_{ij}$$
$$Player\ B's\ Payoff\ = \alpha_{ij}$$

The above results are obtained from zero-sum property of two-person zero-sum games which means that the payoff to B together sum up to zero.

$$(\alpha_{ij}) + (-\alpha_{ij}) = 0$$
$$Or$$
$$(-\alpha_{ij}) + (\alpha_{ij}) = 0$$

Therefore in any two-person zero-sum game A's gain is B's loss and vice versa.

3.2 nXm POSSIBLE PAYOFFS REPRESENTATIONS

From the example given above, there are nXm possible payoffs, represented by table 1 below:

Table 1: nXm Possible Payoffs

	B_1	B_2	B_3		B_5
A_1	α_{11}	α_{12}	α_{13}		α_{1n2}
A_2	α_{21}	α_{22}	α_{23}		α_{2n}
A_3	α_{31}	α_{32}	α_{33}		α_{3n}
A_m	α_{m1}	α_{m2}	α_{m3}		α_{mn}

Let us assume the payoffs to player A and player B in two-person-zero-sum game are shown in table 2 below:

Table 2: Payoffs to player A and player B in two-person-zero-sum game.

	B_1	B_2	B_3	B_4
A_1	28	22	18	21
A_2	26	23	24	25
A_3	16	21	25	26

We are supposed to use two tables, one to represent the payoffs to player A and the other to represent payoffs to player B but the convention is to show the payoff to A knowing that it is also the loss to B. This does not imply, however, that A "always wins" and B "always loses"

We assume that both players know the whole payoff table shown in table 2 above. They know not only the possible payoffs to themselves but equally well they know those of their opponents.

In table 2 above, player B has n = 4 courses of action while player A has m = 3 course of action. The payoffs to A happen to be all positive numbers. The table shows that A will gain something between a minimum of 16 and a maximum of 28. Player B will lose the corresponding quantity. However, the exact size of this transfer of value from B to A is determined by the decisions of both players.

3.3 NATURE OF A SOLUTION

Game theory answers two questions:

i) How will the players behave?

ii) How should the players behave?

These two questions must be answered simultaneously. Let us consider the viewpoint of one decision maker. Player *A* must decide how he should behave. In doing this, player *A* set his objectives as the maximisation of his expected gain. But player *A's* gain is determined not only by his choice of action but also by that of player *B*. Therefore, player *A* cannot decide how he should behave without simultaneously deciding how player *B* will behave.

Main problem of game theory is to determine what the opponent will do. Decision maker A needs a model of his opponent, decision maker B. We now have two choices:

a) modeling his opponent as a chance mechanism and assigning probabilities to the possible action B.

b) modeling the opponent as an intelligent, knowledgeable individual acting in pursuit of his own interests, a "rational" opponent.

The first choice will lead to a problem of decision under uncertainty while the other one is a more reasonable assumption. Also, it will be seen that the assumption of a rational opponent is a conservative assumption in the sense that a rational opponent is the hardest to play against. An opponent who is simple a chance mechanism could not give a lower expected gain to decision maker A.

It is easier to predict the behaviour of a rational opponent than that of an irrational one therefore, a rational opponenet is the most demanding upon A's expected gain, he is also easier to understand. There is just one way to be rational whereas there are many ways of being irrational. It is a key result of game theory that it is possible to predict, either deterministically or probabilistically what the rational opponent will do and therefore to decide what decision maker A should do to maximize his expected payoff.

3.4 GAME WITH A SADDLE POINT

This type of game has the property that you can predict with certainty what the opponent's course of action will be that is the optimal solution to the game. It is worth noting that not all games possess such a solution.

3.4.1 **Nonsecret Strategy**

3.4.1.1 **A's Best Nonsecret Strategy**

The first step in solving a game problem is to find A's best solution assuming that B would know it in advance and counter it. This is called A's best nonsecret strategy. The reasoning is simple and is represented by rows in table 3 shown below for the game specified above.

If A Selects	B would select	A would Receive
A_1	B_3	18
A_2	B_2	23
A_3	B_1	16

Table 3: A's Best Non Secret Strategy

The first row in table 3 shows that if *A* selects his first course of action and *B* knows this choice in advance, *B* would select this third course of action B_3, to limit his loss to 18. After repeating this reasoning for A_2 and again for A_3, *B* would select second and first course of action, B_2, B_1, to limit his loss to 23 and 16 respectively. *A* can finally select his best non secret strategy. From table 3 above, it is A_2 because A_2 has the greatest payoff for *A*. he knows that *B* will select B_2 and the payoff will be 23.

3.4.1.2 **B's Best Nonsecret Strategy**

Table 4 is used to illustrate B's Best Nonsecret Strategy. The first row in table 4 shows that if B selects his first course of action B_1 and A knows this choice in advance, A would select this first course of action, A_1, to maximize his gain to 28. After repeating this reasoning for B_2, B_3 and again for B_4, A would select second course and third course two times for corresponding actions to maximize his gain to 23, 25 and 26 respectively. B can finally select his best nonsecret strategy. From table 4, it is B_2 because B_2 has the greatest payoff for B. He knows that A will select A_2 and the payoff to will be 23. therefore 23 is the smallest loss attainable from a non secret strategy and it is B's best nonsecret strategy.

Table 4: B's Best Nonsecret Strategy

If *B* Selects	*A* would select	*B* would lose
B_1	A_1	28
B_2	A_2	23
B_3	A_3	25
B_4	A_4	26

3.4.2 **Deterministic Solution**

Selection of courses of action mentioned above between player A and player B is in fact made without knowledge of the opponent's choice.

3.4.2.1 **Compelling Reason for Deterministic Solution**

If game theory can tell player A how he should behave, it must also tell player B how player A will behave i.e. if a solution of this type exists it must be a nonsecret solution. Therefore the best nonsecret solutions, if they coincide, are the best solutions to the game.

For the example given above, the best nonsecret strategies do coincide:

A's Best Nonsecret Strategy is A_2;

expecting B to select B_2.

B's Best Nonsecret Strategy is B_2;

expecting A to select A_2.

Each player expects the other to do what is in fact his best nonsecret strategy. When these coincide, the game is said to have a saddle-point or deterministic solution. Deterministic in the sense thst player A can predict with certainty that B will select B_2 if he is confident that the assumptions of the game model are true (that B is rational, that the payoffs in the table are perceived by B to be the correct payoffs, and so forth). If there is no point where the two best nonsecret strategies coincided, then there does not exist a deterministic solution.

A saddle-point or deterministic solution exists if one cell in the table is the smallest entry in its row and simultaneously the largest entry in its column. The efficient method of finding saddle-point solution is

i) Find the smallest entry for each row and mark it with a B because it is B's best countermove if B knew that A would select that row. It is possible for smallest entry not to be unique in row considered therefore, all the entries that are the smallest entry must be marked with B.

ii) In each column, find the largest entry and mark it with an A because it is A's best countermove if A knew that B would select that column. If the largest entry is not unique, mark all equal ones with A.

iii) if at least one entry has been marked with both an A and a B, it is a sddle-point or deterministic solution. If no entry is marked twice, there exists no saddle-point solution. It means that player A cannot predict with certainty what B will select and he is not confident that the assumptions of the game model are true. Table below illustrate the above in details.

	B_1	B_2	B_3	B_4
A_1	28^A	22	18^B	21
A_2	26	$_A23^B$	24	25
A_3	16^B	21	25^A	26^A

Table 5: Calculations for Saddle-Point or Deterministic Solution.

In table 5, there is a saddle-point (deterministic) solution. A will select A_2 and B will select B_2 and the payoff will be a transfer of 23 units from B to A. This model accurately describes the problem and can confidently predict the outcome and the value of the game.

3.4.2.2 **Communication Between the Players Before the Game**

Some communication allowed prior to the final decision and determination of outcome. Each player attempts to test the rationality of his opponent by bluffing. For example, player *B* may lead off with the statement that "I plan to select B_1". He reasons that if *A* believed him, *A* would plan to select A_1. If B believes that *A* believes his statement, he will pick B_3 to reduce his loss to 18. However, it is in *A's* interest to pretend to believe *B's* statement because, if he appears to believe it, he can expect *B* to select B_3. Meanwhile *A* can plan to select A_3 and expect a payoff of 35. There is no equilibrium to this chain of reasoning. Neither player can be confident's that his opponent will do as he says, because it would be against that opponent's self-interest. The only credible communication that *B* could make is "I plan to select B_2". However, that communication is unnecessary because *A* already known that a rational opponent will select B_2. Thus it appears that the possibility of communication has no effect upon the solution of the game if each player believes in the rationality of his opponent.

3.5 CHANCE MECHANISM

Consider the following game in table ^ below in which both decision-makers A and B have two possible courses of action.

	B_1	B_2
A_1	$^{A}4_{B}$	5^{A}
A_2	2	1^{B}

Table 6: Game against a chance

The search for a saddle-point in table 6 reveals that the best nonsecret strategy of each player to select his first course of action.

Let us assume that decision maker A wishes to model his opponent as a chance mechanism rather than a rational opponent. This means he will specify a probability y_1 that the chance mechanism will select course of action B_1. Let y_2 represent the probability that the chance mechanism will select B_2. These two events are mutually exclusive and are the only possible outcomes. Therefore the probabilities must obey

$$y_1 + y_2 = 1 \qquad or \qquad y_2 = 1 - y_1$$

Considering table 6 obove:

The expected payoff of this game against chance, if A selects A_1, is

$$4y_1 + 5(1 - y_1).$$

If A selects A_2, his expected payoff is

$$2y_1 + (1 - y_1).$$

For example, if decision maker *A* believes that chance is equally likely to select B_1 or B_2, the expected values are 4.5 for A_1 and 1.5 for A_2. He would select A_1 and expect to gain 4.5

From the above, if B is a rational opponent then A can expect the payoff of 4, whereas he can expect a payoff of 4.5 if his opponent is a chance mechanism having $y_1 = \frac{1}{2}$. Apparently it is worse to play against a rational opponent. The chance mechanism cannot be a worse opponent then a rational opponent for any value of y_1. This is true because

$$4y_1 + 5(1-y_1) > 2y_1 + (1-y_1) \quad \textit{for any} \quad 0 \le y_1 \le 1$$

and therefore A would select A_1 for any value of y_1. Also, the smallest possible value of

$$4y_1 + 5(1-y_1)$$

is the value 4, chance mechanism has $y_1 = 1$. Thus, the worst possible chance mechanism has $y_1 = 1$ and therefore behaves just like a rational opponent who would also select B_1 with probability 1. Thus a rational opponent is like the least favorable chance mechanism all other chance mechanism $y_1 < 1$, would be a more desirable opponent.

To find the least favourable chance mechanism for this game is a two-variable linear program. Let v represent the expected payoff to player A (and expected loss to player B). Then

Minimize v

Subject to

$$4y_1 + 5(1-y_1) \le v$$
$$2y_1 + (1-y_1) \le v$$

$$y_1 \le 1$$

and

$$y_1 \ge 0$$

3.6 GAMES WITHOUT A SADDLE POINT

Many games do not have a saddle-point solution. However, they still have a solution. Let us consider the game in table 7 below:

	B_1	B_2
A_1	-5_B	6^A
A_2	4^A	-3^B

Table 7: Game without a Saddle Point

A search for a saddle point quickly shows that there is none. Considering decision maker A's viewpoint, if game theory told him to select A_1, it would also tell his opponent that he will select A_1. Then B would select B_1, the least favourable outcome to A. if it told him to select A_2, then B would know to select B_2. There would be no way for A to obtain the favourable outcomes of this game because the information it would give to B about A's decision can always be used to B's advantage.

If there is to be a solution to this game, it must tell A how to decide without simultaneously telling B what A will decide. A solution must have the property that it is a procedure to make a decision while protecting the information about what the decision will be. The only way to accomplish this is by chance mechanism called a random strategy.

3.6.1 **Random Strategy**

Game Theory can tell decision maker *A* to select A_1 with probability x_1 and select A_2 with probability x_2 (such that $x_1 + x_2 = 1$). This tells A how to decide without telling B what decision will be made.

This can be implemented operationally by using any standard chance mechanism such as a random number table. A random number between zero and 1 will be drawn by chance from the table. If the number is less than x_1, then A_2 is selected. This is a superior solution compared to any deterministic solution because it gives decision maker A a higher expected payoff. Player B can know these probabilities but is denied perfect predictability of decision maker A's action.

3.6.1.1 **Best Nonsecret Random Strategy**

Game Theory determines the best values of x_1 and x_2, *A's* best nonsecret random strategy. When *A* has just two possible courses of action, as in this example, there is just one unknown to be determined x_1. Since the two probabilities must sum to 1. x_2 can be found by $x_2 = 1 - x_1$. The method of solution is to consider the expected payoff to *A* of each of *B's* possible courses of action as function of x_1, which is the decision variable of this problem. If *B* picks B_1, the expected payoff to *A* is

$$-5x_1 + 4(1 - x_1)$$

If B picks B_2, the expected payoff to A is

$$6x_1 - 3(1 - x_1)$$

Player B will know the x_1 value in advance and select B_1 or B_2 depending upon which of these expected payoffs to A is smaller for the given x_1 value. Figure 1 shows the graph of these two functions of x_1. The line marked B_1 in figure 1 is the expected payoff to A if B selects B_1. Similarly B_2 is the expected payoff to A if B selects B_2. it is worth nothing that for any particular value of x_1, one line is lower than the other. There is one point where both lines have the same height. This is found by equating the expected payoffs of the courses of action and solving for x_1.

$$-5x_1 + 4(1 - x_1) = 6x_1 - 3(1 - x_1)$$
$$4 - 9x_1 = -3 + 9x_1$$
$$x_1 = 7/18 = 0.388$$

For $x_1 < 0.388$, the line B_2 is lower and so B would select B_2 for all such x_1 values. For $x_1 > 0.388$, the line B_1 is lower and so B would select B_1 for all such x_1 values The expected payoffs to A from B's for best counterstrategy is the line B_2 for $0 \le x_1 < 0.388$ and the line B_1 for $0.388 < x_1 \le 1$. Clearly 0,388 is the best value for x_1 and it is the greatest expected payoff to A. thus A's best nonsecret random strategy is $x_1 = 0.388$ and $x_2 = 1 - 0.388 = 0.612$

3.6.1.1.1 **Optimum Nonsecret Random Strategy**

From example given in table 7 and analysed above, let v denote the expected payoff to A of the optimum nonsecret random strategy. This is found from the height of either line at $x_1 = 0.388$

$$-5x_1 + 4x_2 = v$$
$$\Rightarrow v = -5(0.388) + 4(0.612) = 0.50$$

And

$$6x_1 - 3x_2 = v$$
$$\Rightarrow v = 6(0.388) - 3(0.612) = 0.50$$

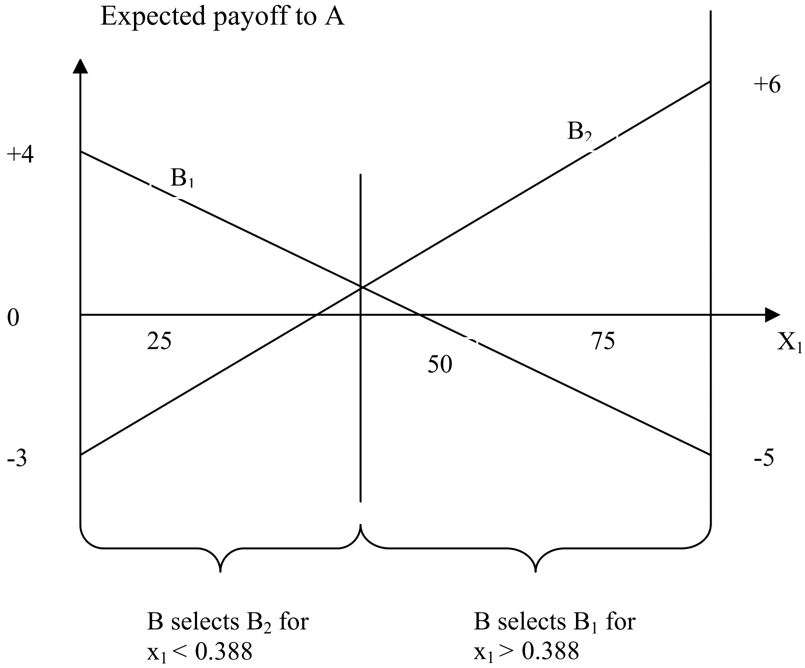

Figure 3.1: Expected Payoff to A

If there had been no intersection of the two curves for an x_1 value in (0.1), one of B's courses of action would have been best for all $x_1 = 0$ or $x_1 = 1$ (or, if the line is level, at both). This implies that a saddle-point solution exists.

To solve for B's best nonsecrete random strategy while preventing his opponent from knowing his actual decision in advance. His strategy will be represented by y_1, the probability that he will select course of action B_1. Then the probability that he will select B_2 is $y_2 = 1 - y_1$. the easiest, most direct way to find y_1 comes from the most direct way to find y_1 comes from the knowledge that the expected loss to B if A picks A_1 is

$$-5y_1 + 6y_2$$
$$\Rightarrow -5y_1 + 6(1 - y_1)$$

Now the expected loss to B is also the expected gain to A; which is known to be 0.50. Therefore y_1 can be determined from

$$-5y_1 + 6(1 - y_1) = 0.50$$
$$5.5 = 1.1y_1$$
$$y_1 = 0.50$$

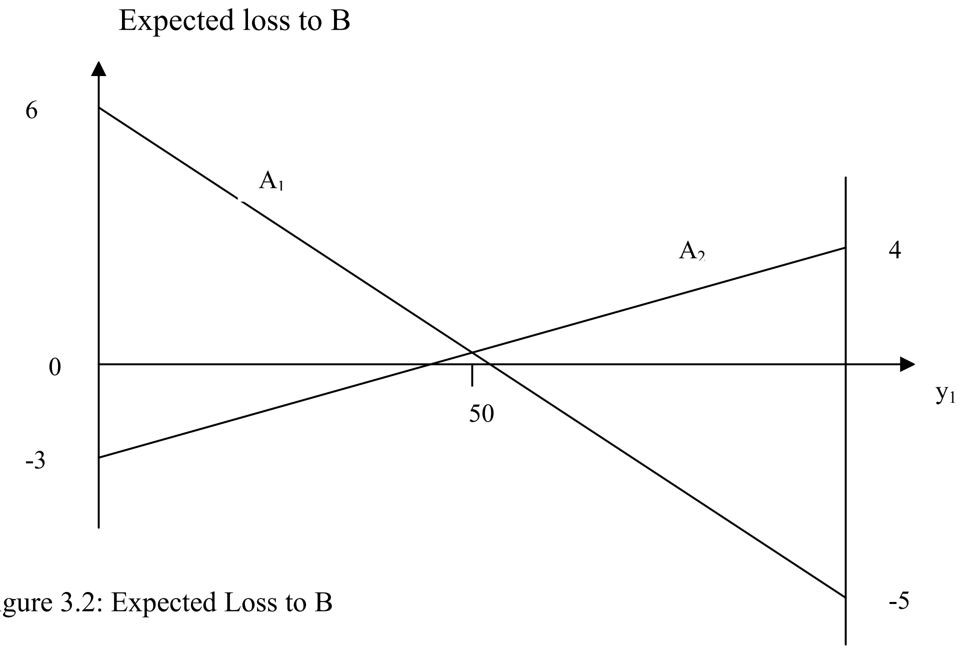

Figure 3.2: Expected Loss to B

From figure 2 above, B’s Best nonsecret random strategy is $y_1 = 50$, $y_2 = 50$ and his expected loss is 50.

To emphasize the symmetry of the problem, the expected loss to B is

$$-5y_1 + 6(1 - y_1) \text{ if A selects } A_1$$

and

$$4y_1 - 3(1 - y_1) \text{ if A selects } A_2.$$

These are graphed as a function of y_1 in figure 2 above. The two expressions are equated to solve for the y_1 point where the lines cross:

$$-5y_1 + 6(1 - y_1) = 4y_1 - 3(1 - y_1)$$

The solution is

$$y_1 = 0.50$$

The expected loss to B is also v

$$v = -5(0.50) + 6(0.50) = 0.50$$

3.7 **REDUCTION OF THE SIZE OF A GAME**

If one course of action is better than or as good as another for all possible courses of action of the opponent, then the first is said to “dominate” the second course of action. The dominated course of action can be simply discarded because it is of no value. This idea can be used to reduce the size of a game. However, this useful only when the game does not have a saddle point because a saddle point, when present, is easy to find. When there is no saddle point present it is important to try to reduce the size of the game by dominance. Let us consider the game in table 8a below:

	B_1	B_2	B_3	B_4	B_5
A_1	4	4	2	-4	-6^B
A_2	8	6^A	8^A	-4^B	0
A_3	10^A	2^B	4	10^A	12^A

Table 8a: Demonstration of Dominance

A search has shown that no saddle point is present. A search for dominance shows that B_2 dominates B_1 and so B_1 can be discarded. The new table is shown below in table 8b.

	B_2	B_3	B_4	B_5
A_1	4^A	2^B	-4	-6^B
A_2	6	8^A	-4^B	0
A_3	2^B	4	10	12^A

Table 8b: A search for Dominance (B_1 and B_2 compared)

A search of table 8b reveals that A_2 dominates A_1. A_1 can be discarded to give table 8c below:

	B_2	B_3	B_4	B_5
A_2	6	8	-4	0
A_3	2	4	10	12

Table 8c: A search for Dominance (A_1 and A_2 compared)

There may be new dominance relationships in table 8c above. These relationships were not present before. For example, B_2 now dominates B_3 whereas it did do so in table 8a. Similarly, B_4 now dominates B_5. Both B_3 and B_5 are discarded to give table 8d below:

	B_2	B_4
A_2	6	-4
A_3	2	10

Table 8d: Final dominance relationship

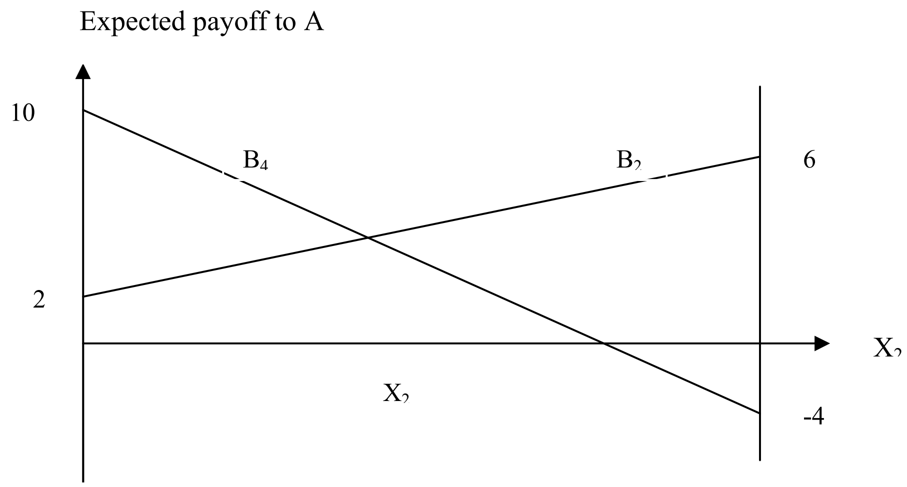

Figure 3.3: Expected payoff to A (dominance)

Thus dominance has reduced a 3x5 game to a 3x3 game that can be solved by the method used under game without saddle point above. Using the method, it is known already that $x_1 = 0$. Therefore $x_2 + x_3 = 1$. It remains to solve for x_2, A's best nonsecret random strategy. It can be found as follows. If B selects B_2, the expected payoff to A is

$$6x_2 + 2(1 - x_2)$$

If B selects B_4, it is

$$-4x_2 + 10(1 - x_2)$$

The graph is shown in figure 3 above. The equation that determines the best x_2 is

$$6x_2 + 2(1 - x_2) = -4x_2 + 10(1 - x_2)$$

The solution is

$$x_2 = 4/9$$

Therefore $x_3 = 5/9$ from $(x_3 = 1 - x_2)$

The expected payoff to A is

$$6x_2 + 2x_3 = 6(4/9) + 2(5/9)$$
$$= 34/9$$
$$= 3.78$$

The best nonsecret random strategy for B has $y_1 = 0$, $y_3 = 0$, $y_5 = 0$ and $y_2 + y_4 = 1$.

The value of y2 can be obtained from $6y_2 - 4(1 - y_2) = 34/9$

The solution is $y_2 = 7/9$. Therefore $y_4 = 2/9$. The graph is shown in figure 4 below:

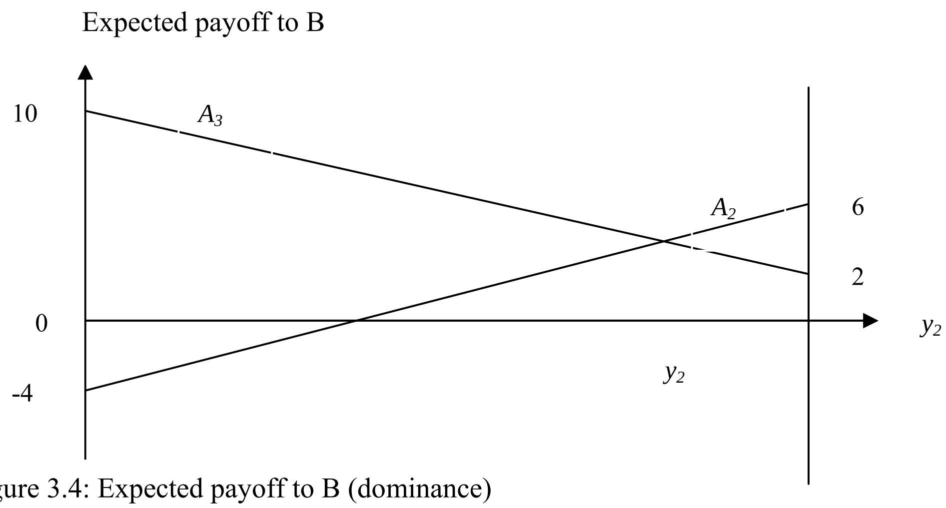

Figure 3.4: Expected payoff to B (dominance)

The equation that determines the best y_2 in the figure 4 above is

$$6y_2 - 4(1 - y_2) = 2y_2 + 10(1 - y_2)$$

The solution is

$$6y_2 - 4 + 4y_2 = 2y_2 + 10 - 10y_2$$
$$10y_2 - 4 = 10 - 8y_2$$
$$18y_2 = 14$$
$$y_2 = 14/18 = 7/9 \quad (Q.E.D)$$

Therefore

$$y_4 = 2/9$$

The expected loss to B is

$$6y_2 - 4y_4 = 6(7/9) - 4(2/9) = 61/9 - 38/9$$
$$= 23/9 = 2.56$$

3.8 **2Xn GAMES**

The difficulty in solving a game that has no saddle – point solution is determined by the smaller of the game's two dimensions. If A has only two courses of action, the statement implies that B also will use no more than two. This implies that all 2Xn games can be easily solved by the methods of games without a saddle point solution. Considering the following table:

	B_1	B_2	B_3	B_4	B_5
A_1	4	-2	10	-4	12
A_2	-4	8	-6	2	0

Table 9: 2Xn Game

A's best nonsecret random strategy (x_1,x_2) is described by one number, x_1. B's best nonsecret random strategy is describe by four numbers, y_1,y_2,y_3, and y_4, where y_5 is determined by

$$y_5 = 1 - y_1 - y_2 - y_3 - y_4$$

To determine x_1 it is necessary to consider all of B's possible choices. The expected payoff to A is

$$4x_1 - 4(1 - x_1) \quad If \;\; B \; selects \; B_1$$
$$-2x_1 + 8(1 - x_1) \quad If \;\; B \; selects \; B_2$$
$$10x_1 - 6(1 - x_1) \quad If \;\; B \;\; selects \;\; B_3$$
$$-4x_1 + 2(1 - x_1) \quad If \;\; B \;\; selects \;\; B_4$$
$$12x_1 = 0(1 - x_1) \quad If \;\; B \;\; selects \;\; B_5$$

These can be graphed as a function of x_1 as shown in figure 5 below.

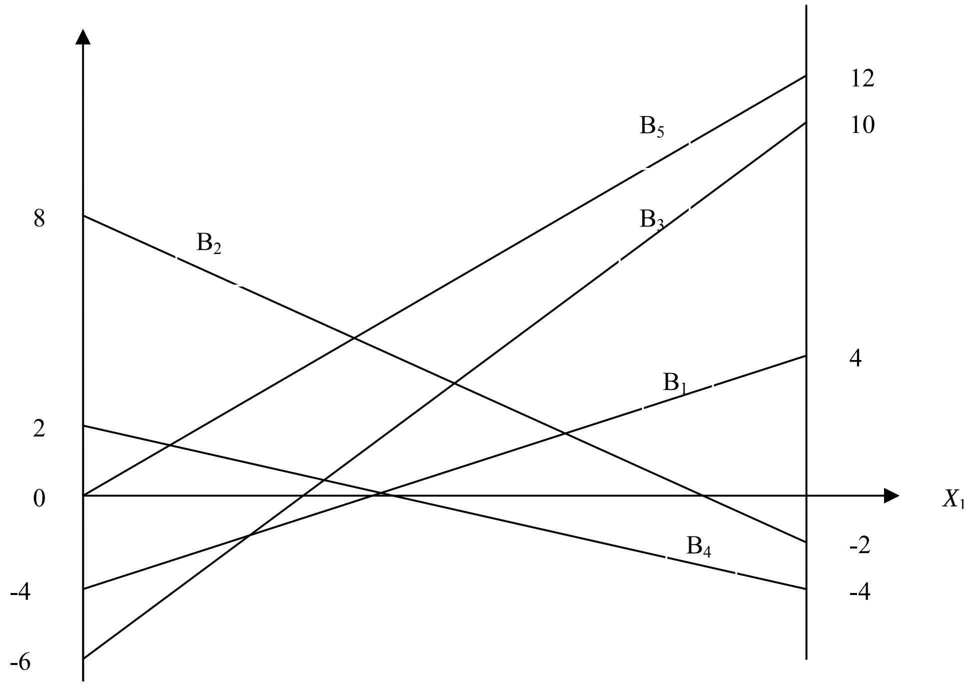

Figure 3.5: Expected Payoff to A

When the graph is drawn into scale, it is immediately clear that for x_1 values near zero, B would select B_3, for intermediate values he would select B_1, and for values near 1 he would select B_4. He would never selected B_2 or B_5. This could have been determined by the use of dominance as earlier explained. By dominance method B_2 is dominated by B_4 and B_5 is dominated by B_3 as shown in table 9. Therefore they can be discarded and both y_2 and y_5 can be set to zero.

The next observation from figure 5 is that the intersection of B_1 with B_4 is higher than the intersection of B_1 with B_3. Therefore it is the intersection of B_1 with B_4 that determines x_1:

$$4x_1 - 4(1-x_1) = -4x_1 + 2(1-x_1)$$

$$x_1 = 3/7$$

And so

$$x_2 = 4/7$$

The expected payoff to A is

$$v = 4(2/7) - 4(4/7) = -4/7$$

A further conclusion is that B will never use B_3. his best random strategy will use only B_1 and B_4.

To find B's best random strategy is now easy. It is known that B_2, B_3, and B_5 will not be used. Therefore y_2, y_3, and y_5 are zero. That leaves y_1 and y_4 to be determined. Since $y_1 + y_4 = 1$, there remains only y_1 to be determined. If A selects A_1, the expected loss to B is $4y_1 - 4(1 - y_1)$ and this must equal the expected payoff to A, which is $-4/7$.

Therefore $4y_1 - 4(1 - y_1) = -4/7$

The solution for y_1 is $y_1 = 3/7$. Therefore $y_4 = 4/7$ and the solution is complete.

In conclusion, when A has only two course of action an optimum solution for x_1 is always at the intersection at one point, only two of them would be used. A similar reasoning would apply to 3Xn games. However, the graphic method used so far is convenient only far two courses of action. For games whose smallest dimension is greater than 2 and cannot be reduced by dominance, a new method must be considered that is generally useful for any dimensions.

3.9 SOLUTION OF GAMES BY LINEAR PROGRAMMING (MANUAL METHOD)

A two-person-zero-sum game generally implies that a has m courses of action (A_1, A_2, …, A_m) and B has n courses of action (B_1, B_2, …., B_n) where m and n are not necessarily equal. If player A selects A_i and player B selects B_j, also α_{ij} and the loss to B is also α_{ij}.

From this example given, A's best random strategy is specified by $x_1, x_2, \cdots, x_m$ while B's best random strategy is $y_1, y_2, \cdots, y_n$. Determining A's best random strategy can be formulated as a their program with $m+1$ decision variables ($x_1, x_2, \cdots, x_m$) and v, where v is the expected payoff to A:

Maximize v

Subject to

$$
\begin{aligned}
&\alpha_{11}x_1 + \alpha_{21}x_2 + \alpha_{31}x_3 + \cdots + \alpha_{m1}x_m \geq v \\
&\alpha_{12}x_1 + \alpha_{22}x_2 + \alpha_{32}x_3 + \cdots + \alpha_{m2}x_m \geq v \\
&\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\
&\alpha_{1n}x_1 + \alpha_{2n}x_2 + \alpha_{3n}x_3 + \cdots + \alpha_{mn}x_m \geq v
\end{aligned}
$$

And $x_1 + x_2 + x_3 + \cdots + x_m = 1$

$$x_1, x_2, x_3, \cdots, x_m \geq 0$$

$$or$$

$$0 \leq x_i \leq 1$$

There are n equality constraints in the above illustration. The j[th] constraint states that the expected payoff to A if B selects B_j cannot be less than v because B will pick his best counterstrategy. The last constraint is an equality constraint: The probabilities must sum to 1. The x's must be nonnegative because they are probabilities. However, the variable v is not restricted to be nonnegative standard simplex method given above constraints all variables to be nonnegative. It is not a problem when all α_{ij} are nonnegative because v cannot then be negative. Therefore the nonnegative problem can be prevented by adding a constant c to all cells in the α_{ij} table such that the resulting table contains all nonnegative entries. This c would be subtracted from the final v value as a last step to recover the true v, which may be negative. The equality constraint can be used for substitution to eliminate one variable if the nonnegative of the eliminated variable is enforced by a constraint.

The solution for $x_1, x_2, \cdots, x_m$ and v can be used to obtain the solution for $y_1, y_2, \cdots, y_n$. Suppose that the first $r \leq m$ of the x's are greater than zero and others are zero. Then the y's can be determined as the solution of simultaneous linear equations. If the j[th] constraint is satisfied as equality in the optimal solution, then $y_j > 0$, this means that B may use Bj. If the j[th] constraint is satisfied as equality in the optimal solution, then $y_j = 0$ and B would never use B_j. this reasoning determines which r of the n. the y values must be solved for also. To solve for B's best random strategy as a linear programming problem:

Minimize v

Subject to

$$\begin{aligned}
&\alpha_{11}x_1 + \alpha_{21}x_2 + \alpha_{31}x_3 + \cdots + \alpha_{m1}x_m \geq v \\
&\alpha_{12}x_1 + \alpha_{22}x_2 + \alpha_{32}x_3 + \cdots + \alpha_{m2}x_m \geq v \\
&\vdots \qquad\qquad \vdots \qquad\qquad \vdots \\
&\alpha_{1n}x_1 + \alpha_{2n}x_2 + \alpha_{3n}x_3 + \cdots + \alpha_{mn}x_m \geq v
\end{aligned}$$

And $$x_1 + x_2 + x_3 + \cdots + x_m = 1$$

$$x_1, x_2, x_3, \cdots, x_m \geq 0$$

$$or$$

$$0 \leq x_i \leq 1$$

From the above, the j[th] inequality represents the expected loss to B if A select A_j. It is worth noting that of the game because the value of the game is determined by A's selection of the best minimize v, his expected loss. This linear program is used to find the probabilities $y_1, y_2, \cdots, y_n$ that are least favourable to player A. this linear program is solved first, then its results, knowledge of which variables are positive and which constraints hold as equalities, are used to solve for A's best random strategy as a set of linear equations rather than by solving A's linear program.

3.9.1 **Formulation of Linear Program for a Specific Problem**

Consider the following example. The two players A and B must select a number out of 1, 2, or 3. If both have chosen the same number, A will pay B the amount of the chosen number. Otherwise A receives the amount of his own number from B. The payoff table for this game is shown in table 10 below.

B

A		1	2	3
	1	-1_B	1	1
	2	2	-2^B	2^A
	3	3^A	3_A	-3^B

Table 10: Payoff Table for Linear Programming

A search indicates that there is no saddle-point solution. The linear program for A's best random strategy, x_1, x_2, x_3 is

Maximize v

Subject to

$$-x_1 + 2x_2 + 3x_3 \geq v$$
$$x_1 - 2x_2 + 3x_3 \geq v$$
$$x_1 + 2x_2 - 3x_3 \geq v$$
$$x_1 + x_2 + x_3 = 1$$

and

$$x_1, x_2, x_3 \geq 0$$

In this case, the optimal v could be negative because the payoff table does contain negative payoffs. However, if the number 3 is added to all α_{ij}'s, then all will be nonnegative. Then define $v' = v + 3 \Rightarrow v = v' - 3$. The problem can be rewritten with all nonnegative variables as

Maximize $v' - 3$

Subject to

$$2x_1 + 5x_2 + 6x_3 \geq v'$$
$$4x_1 + x_2 + 6x_3 \geq v'$$
$$4x_1 + 5x_2 \geq v'$$
$$x_1 + x_2 + x_3 = 1$$

It is best with hand calculation to eliminate the equality constraint by substituting $x_3 = 1 - x_1 - x_2$ and replacing the nonnegative condition $x_3 \geq 0$ by $x_1 + x_2 \leq 1$.

The resulting problem is

Maximize $v' - 3$

Subject to

$$4x_1 + x_2 + v' \leq 6$$
$$2x_1 + 5x_2 + v' \leq 6$$
$$4x_1 + 5x_2 - v' \geq 0$$
$$x_1 + x_2 \leq 1$$

and

$$x_1, x_2, v' \geq 0$$

Slack variables are then used to convert the four inequality constraints to equalities. The problem becomes

Maximize $v' - 3$

Subject to

$$4x_1 + x_2 + v' + s_1 = 6$$
$$2x_1 + 5x_2 + v' + s_2 = 6$$
$$4x_1 + 5x_2 - v' - s_3 = 0$$
$$x_1 + x_2 + s_4 = 1$$

The initial Basic Feasible Solution (BFS) will have basic variables S_1, S_2, S_3, S_4 and zero variables x_1, x_2, v'. This corner point is degenerate because the third constraint passes through the origin.

This first tableau of the simplex method is

Pivot

Const.	x_1	x_2	v'	Ratio
-3	0	0	1	
6	-4	-1	-1	-6
6	-2	-5	-1	-6
0	4	5	-1	0
1	-1	-1	0	α

The next tableau shows no gain in objective because of the degeneracy

	Const.	x_1	x_2	S_3	Ratio	
E	-3	+4	+5	-1		
S_1	6	-8	-6	+1	-1	
S_2	6	-6	-10	+1	-6/10	Pivot
v'	0	+4	+5	-1	0	
S_4	1	-1	-1	0	-1	

The next tableau is

	Const	x_1	S_2	S_3	Ratio	
E	0	1	$-\frac{1}{2}$	$-\frac{1}{2}$		
S_1	22/10	$-44/10$	$+6/10$	$+4/10$	$-24/44$	Pivot
x_2	$+6/10$	$-6/10$	$-1/10$	$+1/10$	-1	
v'	3	1	$-\frac{1}{2}$	$-\frac{1}{2}$	+3	
S_4	$+4/10$	$-4/10$	$+1/10$	$-1/10$	-1	

The next tableau gives the optimal solution

	Const	S_1	S_2	S_3	Ratio
E	6/11	$-10/44$	$-4/11$	$-9/22$	
x_1	6/11	$-10/44$	6/44	1/11	-1
x_2	3/11	$+6/44$	$-2/11$	1/22	6/10
v'	39/11	$-10/44$	$-4/11$	$-9/22$	0
S_4	2/11	$+4/44$	$+\frac{1}{2}$	$-7/110$	-1

The result is

$$x_1 = 6/11$$
$$x_2 = 3/11$$
$$x_3 = 2/11$$

and

$$v = 6/11$$

The above result shows that the first three constants hold as equalities in the optimal solution. Therefore it is known that B uses all three of his courses of action. The best random strategy for B can be found from these results by solving three simultaneous equations in three unknowns. The i^{th} equation represents the expected loss to B if A uses A_i. All these must equal the expected gain to A that is known. The equations are:

$$-y_1 + y_2 + y_3 = 6/11$$
$$2y_1 - 2y_2 + 2y_3 = 6/11$$
$$3y_1 + 3y_2 - 3y_3 = 6/11$$

The solution is

$$y_1 = 5/22$$
$$y_2 = 4/11$$
$$y_3 = 9/22$$

This completes the solution to the game problem.

3.9.1.1 **Analysis of the Game**

First find B's optimal strategy. The linear program is

Minimize v

Subject to

$$-y_1 + y_2 + y_3 \le v$$
$$2y_1 - 2y_2 + 2y_3 \le v$$
$$3y_1 + 3y_2 - 3y_3 \le v$$
$$y_1 + y_2 + y_3 = 1$$

and

$$y_1, y_2, y_3 \ge 0$$

Again v could be negative because some α_{ij} are negative. This problem is circumvented by adding 3 to all payoffs and defining $v' = v + 3$. Also the substitution $y_3 = 1 - y_1 - y_2$ is made and the constraint $y_1 + y_2 \le 1$ is added to ensure the non-negativity of the substituted variable. The problem becomes:

Minimize $v' - 3$

Subject to

$$2y_1 + v' \ge 4$$
$$6y_2 + v' \ge 5$$
$$6y_1 + 6y_2 + v' \le 0$$
$$y_1 + y_2 \le 1$$

and

$$y_1, y_2, v' \ge 0.$$

3.10 SOLUTION OF GAMES BY LINEAR PROGRAMMING (COMPUTERIZED METHOD)

A computerized solution to two-person-zero-sum game problem that has a general statement of course of actions for player A and different course of actions for player A and different course of actions for player B is designed and implemented in Conflict Resolution System (CRS). Here, computer is used to get courses of action of each player and it manipulates them to get solution to the problem. It does this with high speed and accuracy. A and B best random strategies are formulated as linear programs with decision variables. From the linear equations got, tableaus are formed and final result computed and printed on the screen.

3.10.1 Information Requirements for the System

Here, we address two important issues of the practical implementation of the system. First, the basic concept of the Information Structure Perspective (I. S. P) will be discussed. Secondly, the Information Usage Perspective (I. U. P) is also going to be discussed.

3.10.1.1 The Information Structure Perspective (I. S. P)

It describes the natural and conceptual relationships among the operational data. However, the state of a perception of the real world can be regarded as a series of distinct but some times related phenomena. These phenomena are described by data. Data correspond to discrete fact about phenomena from which one gains information about the real world. The payoffs to player A and payoffs to player B are the operational data used. The integrated view of the operational data is described as well.

3.10.1.1.1 The Operational Data

The operational data is described by using the relational database model. The relational database model is based on a foundation of theory from relational algebra. Consequently, it is a high level abstraction of a universe of discourse. It consists of a group of concepts that are particularly related to any programming language.

A relation is a matrix or two-dimensional table of related data that has several properties. In this case, it is a two dimensional table of numerical data showing payoffs to both players.

The general form of a relation is given by: $R[A_1, A_2, A_3, \cdots, A_{k-1}, A_k, A_{k+1}, \cdots, A_{n-1}, A_n]$

Where R represents a strategy of a player, the set $\{A_j\}$, $j = 1, 2, 3, 4, \cdots, n$ represents the payoffs to the player. In the case of players, A and B, it is noted that where the payoffs to A equals the expected loss by B is called saddle-point. The payoff tables described above are identified with the computer aided system.

3.10.1.1.2 **The Integrated View of the Operational Data**

The integrated view of the payoffs is shown in figure 7 below:

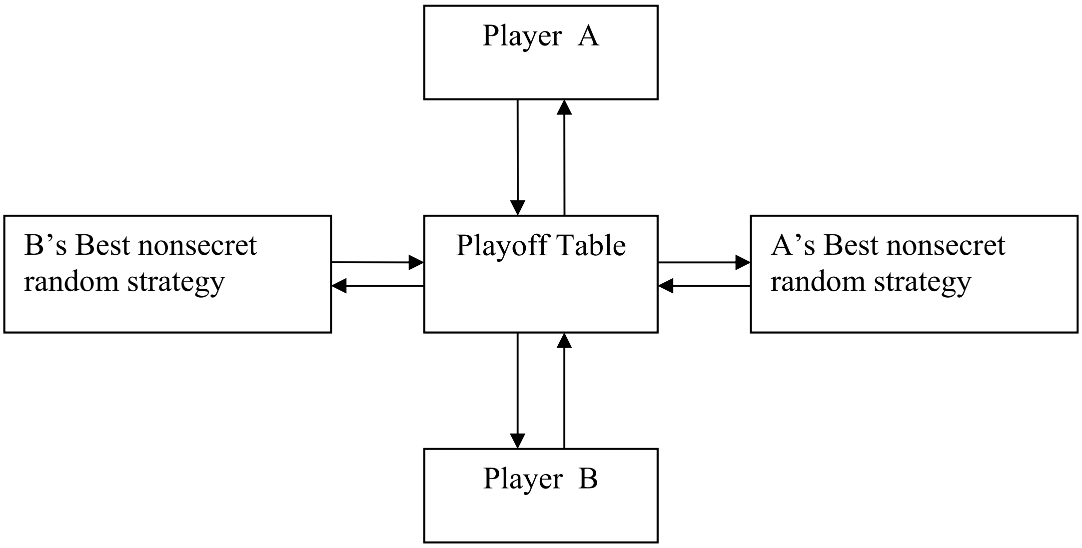

Figure 3.6: Integrated View of the Operational Data.

The entity types in the above figure functionally or logically dependent on each other.

3.10.1.1.3 **The Data Constraints**

The database model defines the rates which bind the logical relationships and constraint among the payoffs. It gives adequate interpretation of the meaning of payoff and how they are being used. In the real sense, the constraints binding on the system can be classified into two categories namely:

a) Integrity Constraint
b) Semantic Constraint

The integrity constraints are concerned with the areas of the system, which are applied to individual payoff. They are also concerned with the rules that bind the values of payoffs.

A semantic constraint is concerned with the rules, which bind the meaning of payoff with a view to reflecting the naturalness of payoff representation.

Database rules provide the mechanism, which enforced standard and central control.
The semantic constraints of the operational data of game theory are spelt out below:

i) Repetition is not allowed.

ii) Just one play and then the problem is over.

iii) Decisions of both players are made individually, prior to the play.

iv) No communication between the players.

v) Decisions are made simultaneously so that neither player has an advantage resulting from direct knowledge of the other's decision.

Transaction

The game involves only two players. It also involves direct conflicts between the two players. The game is played once and the problem is over. At the end of the game one of the players will gain while the other loses. The gain and the loss equal in magnitude. It means that the payoffs to both when sum together give zero-sum. Some of the transactions involved before arriving at the solution to the game are

i) Construction of payoff table.

ii) Finding saddle point solution.

iii) Finding the best nonsecret random strategy for each player.

iv) Reduction mechanism.

v) Formulation of linear program

3.11 EXPLOITING AN OPPONENT'S MISTAKES

The definition of 'best strategy' implies that the game is played against a rational opponent whose object is to maximise his own gain. It is possible for player A to take advantage of the knowledge that player B is not using his best strategy.

For example, if player A's strategy is $\left(\frac{1}{2}, \frac{1}{2}\right)$ in the game of head or tail, and is optimal against a shrewd opponent in that it protects him against loss. If, however, B is observed to play 'heads' more frequently than 'tails', A can increase his gain by also playing 'heads' more frequently than 'tail'. If, for example, B plays 'heads' twice as often as 'tails', i.e. a strategy of $(2/3, 1/3)$, A increases his gain by also choosing for his strategy $(2/3, 1/3)$. The gain to A under these circumstances is $2/3(1/3) + 1/3(-1/3)$, i.e. $1/9$, player A winning, on average, 5 games out of 9. it can be shown that if the opponent's strategy is known in advance, a player achieves his maximum gain to 1/3 by playing heads through out. Unfortunately, if A did this, B would almost certainly notice and be led t modify his own strategy.

3.12 **GAMES AGAINST NATURE**

We have shown in the previous section how a player may take advantages of knowledge of his opponent's strategy. Many decision-making situations can be viewed as zero-sum two-person games where the opponent is nature . such a situation differs from a game in that Nature is not actively engaged in trying to outwit her opponent. Her behaviour is independent of her opponent's and may well be to some extent predictable. The being so, a player against nature should be able to select that pure strategy which maximizes his expected gain.

In everyday life we are constantly making decisions; some trivial, some not so trivial. Although we rarely, if ever, formulate the decision in detail, we may nevertheless unconsciously carry out a similar exercise. There are decisions, however, where the issues are of such importance as to demand time for quantitative analysis. It is in such situation that game theory is valuable. Its ultimate usefulness in practice depends on how exhaustively the courses of action may be determined and how accurately the possible outcomes may be measured. Game theory does at least provide a framework with which the relevant factors involved in a decision may be isolated.

CONCLUSION

Having examined the manual/traditional and electronic approaches to games and game theory, the introduction of CRS developed into game theory to aid efficiency and to increase the accuracy in analyzing games related problems or finding solution to them will help a long way to eradicate some of the listed problems facing selection of best nonsecret strategies by players, selection of course of actions between players and chance mechanism for decision making between two players. The provision of computer with on-line and real-time processing will help to eradicate these problems in game theory.

CRS (Conflicts Resolution System) has been able to effectively perform the specified software functions as earlier stated. It is able to acquire data, make the necessary manipulations and statistical computation based on the chosen statistical design and generate the necessary and relevant report either by printing the file created or by graphical display or by graphical printout.

The package is multifunctional, flexible and capable of representing solutions to game theory problem in graphical form: It is able to draw Line Graphs for Expected gains to player A and Expected loss to player B, and line graphs for solutions to the game theory by linear program. Also, it can determine saddle_point solution from the payoff table if there is any. In addition, it can perform alternative functions to determine solution to the problem if there is no saddle_point solution.

The system developed helps to eradicate some of the problems/limitations of manual approach. It is important to emphasize that the areas of applications of this system cannot be exhausted in this dissertation. Much achievement has been made in the areas of data security, integrity, confidentiality, constraints and restrictions.

Much effort and time has been put into the development of this system and the logic involved allows for the achievement of the aims and objectives of this project work as well as provision for systems flexibility.

<u>The Contributions of CRS</u>

The implementation of the system will improve the existing manual approach to resolving conflicts among opposing interests by reducing the time of operation.

The contributions of the system include the following:

- Proper keeping of the various games data.
- Reduction in the long time of processing result of conflicts among opposing interests.
- Provision of data integrity and reliability through data validation.
- It is a tool for the intensive training to teach people how to resolve conflicts.

- For proper file maintenance and provision of accurate reports.

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