

Linear Optics Quantum Computation

Produced with pdflatex and xfig

- Optical quantum computation.
- Photon basics.
- On non-linear gates.
- Computation with linear optics.
- Challenges.

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Photonic Qubit

- Photonic qubit: One photon in a superposition of two modes.



- Photonic qubits are usually "flying" qubits.
- Making a superposition state:



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Optical Quantum Computing

- **Advantages.**
 - Easy to observe interference.
 - Room temperature.
 - Fast ops/slow decoherence.
 - Lots of photons.
- **Challenges.**
 - Mode matching.
 - Non-linear interactions.
 - ... except photon sources/detectors.
 - Fast feed-forward needed.
 - Low loss optics.
 - Single photon sources.
 - Photon counters.
- Schemes for optical quantum computation.
 - With non-linearities:
 - Single photon qubits, Kerr non-linearities. Milburn 1988 [1]
 - Optics and cavity QED. Turchette&al. 1995 [2]
 - With exponential resources:
 - Linear optics, one mode per quantum dimension. Cerf&Adami&Kwiat 1998 [3]
 - Primarily linear:
 - eLOQC. Knill&Laflamme&Milburn 2000 [4]
 - Linear optics with squeezed state qubits. Gottesman&Kitaev&Preskill 2000 [5]
 - Linear optics with Gaussian state qubits. Ralph&Munro&Milburn 2001 [6, 7]

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Optical Modes

- Mode visualization.



- Mode A can have 0, 1, 2, ... photons.

$$|0\rangle_A, |1\rangle_A, |2\rangle_A, \dots, |n\rangle_A, \dots$$

State space: Superpositions $\sum_k \alpha_k |k\rangle_A$.

- Mode A operators: $(\mathbf{a}^\dagger)_A : |n\rangle_A \rightarrow \sqrt{n+1} |n+1\rangle_A$
 $(\mathbf{n})_A : |n\rangle_A \rightarrow n |n\rangle_A$

- Bosonic qubit $Q(A, B)$ on modes A, B:

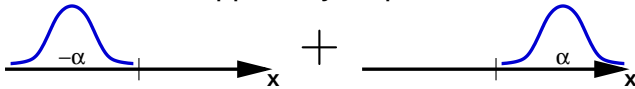
$$|0\rangle_{Q(A,B)} \leftrightarrow |0\rangle_A |1\rangle_B, \quad |1\rangle_{Q(A,B)} \leftrightarrow |1\rangle_A |0\rangle_B$$

Optical network notation for $Q(A, B)$:



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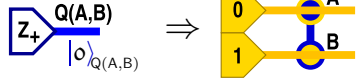
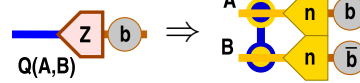
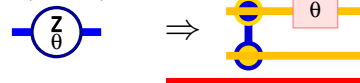
Coherent State Qubits

- Coherent state of amplitude α .
 $|\alpha\rangle = \sum_{k=0}^{\infty} e^{-|\alpha|^2/2} \frac{\alpha^k}{\sqrt{k!}} |k\rangle$, characterized by $\mathbf{a}|\alpha\rangle = \alpha|\alpha\rangle$
- Qubit state identification. Let $\alpha \approx 3$.
 $|0\rangle_Q \mapsto |\alpha\rangle$, $|1\rangle_Q \mapsto |-\alpha\rangle$.
 $\dots$ approximately, since $\langle -\alpha|\alpha\rangle = e^{-2|\alpha|^2} \approx 1.5 \times 10^{-8}$.
- “Cat” state.
 $\frac{1}{\sqrt{2}}(|0\rangle_Q + |1\rangle_Q) \mapsto \frac{1}{\sqrt{2}}(|-\alpha\rangle + |\alpha\rangle)$.
 Superposition of two oppositely displaced Gaussians.

- Scalable q. comp. is possible with cat states, linear optics and photon counters using coherent state qubits.

Ralph&Munro&Milburn 2001 [6] &Gilchrist&Glancy 2003 [7]

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Bosonic Qubit Operations

- Preparation of $|0\rangle_{Q(A,B)}$:

 - Measurement of $(\sigma_z)_{Q(A,B)}$:

 - $Z = \sigma_z$ rotation by θ , $e^{-i\sigma_z\theta/2}$:
 $\propto \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$:

 - $X = \sigma_x$ rotation by ϕ , $e^{-i\sigma_x\phi/2}$:
 $\begin{pmatrix} \cos(\phi) & -i\sin(\phi) \\ -i\sin(\phi) & \cos(\phi) \end{pmatrix}$:

 - $Y = \sigma_y$ rotation by ϕ , $e^{-i\sigma_y\phi/2}$:
 $\begin{pmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi) \end{pmatrix}$:

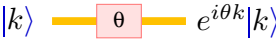
- Still need a “nonlinear” coupling.

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Optical Devices for LOQC

- Mode preparations: 
- Photon counter: 
 $\dots$ can be used for feed-forward.

Passive linear-optics.

- Phase shifter $\mathbf{P}_\theta$:  $e^{i\theta k}|k\rangle$
- Beam splitter $\mathbf{B}_{\theta,\phi}$:

 $|1\rangle_A|0\rangle_B \rightarrow \cos(\theta)|1\rangle_A|0\rangle_B + e^{-i\phi}\sin(\theta)|0\rangle_A|1\rangle_B$
 $|0\rangle_A|1\rangle_B \rightarrow -e^{i\phi}\sin(\theta)|1\rangle_A|0\rangle_B + \cos(\theta)|0\rangle_A|1\rangle_B$
- U is *passive linear* if $U\mathbf{a}_s^\dagger U^\dagger = \sum_r u_{rs}\mathbf{a}_r^\dagger$.
 $U|0\rangle = |0\rangle$, $\hat{u} = (u_{rs})_{rs}$ unitary.

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Linear Optics No-Go?

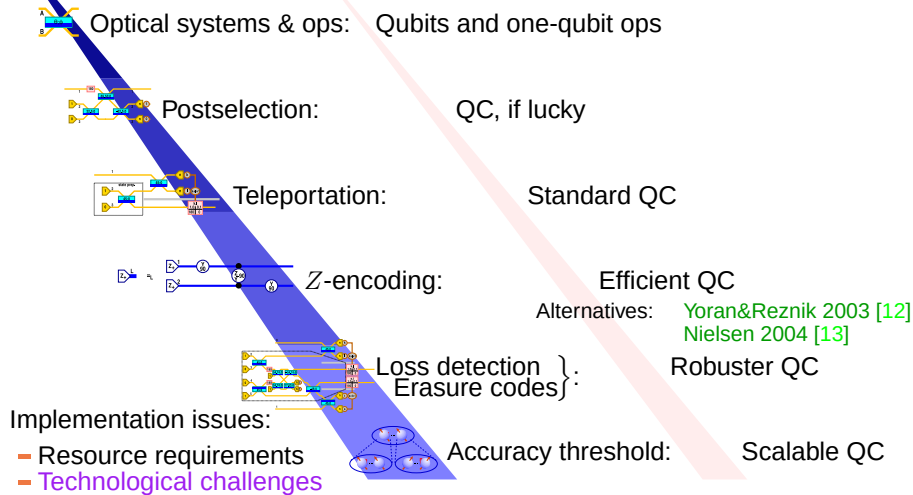
- Passive linear optics $\simeq$ classical wave mechanics?
- Observation.** Coherent state preparations, passive linear optics and photodetectors with feedback are efficiently classically simulatable.
 $\dots$ because modes never become correlated.
- Theorem.** Linear optics and particle detectors for fermions are efficiently classically simulatable.
 Valiant 2001 [8], Terhal&DiVincenzo 2001 [9], Knill 2001 [10]
- Theorem** Linear optics and measurement of $\hat{x} \propto \mathbf{a}^\dagger + \mathbf{a}$ is efficiently classically simulatable.
 Bartlett&al. 2001 [11]
- Theorem** Linear optics, single photon sources and photon counters are sufficient for q. comp.
 Knill&Laflamme&Milburn 2000 [4]

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eLOQC Guide

Optical methods

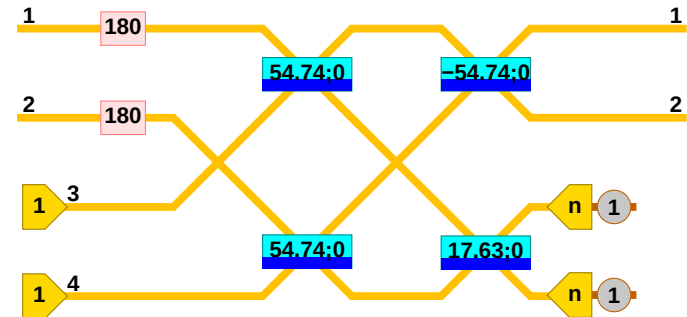
Capabilities realized



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Linear Optical Controlled Sign Flip

- $|ab\rangle_{12} \rightarrow (-1)^{a \cdot b} |ab\rangle_{12}$ with success probability $1/13.5$:

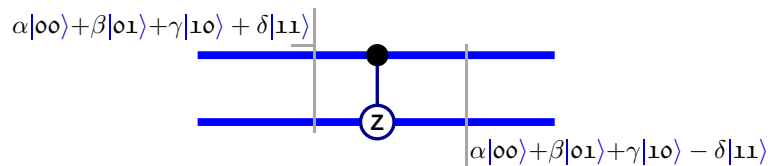


- Q. comp. with exponentially small probability of success.
- Practical application: State preparation.

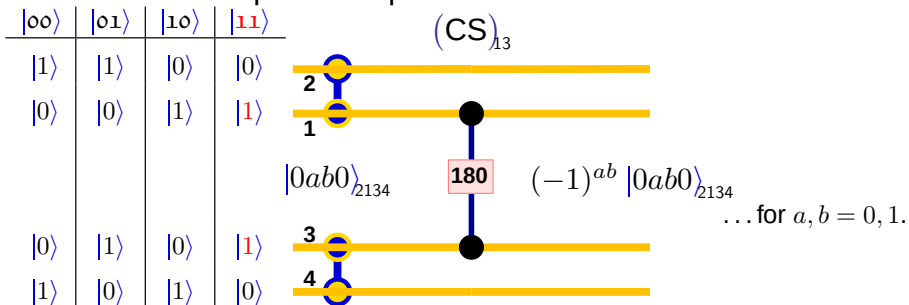
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Controlled Sign Flips

- Sufficient to add sgn , the controlled sign flip.



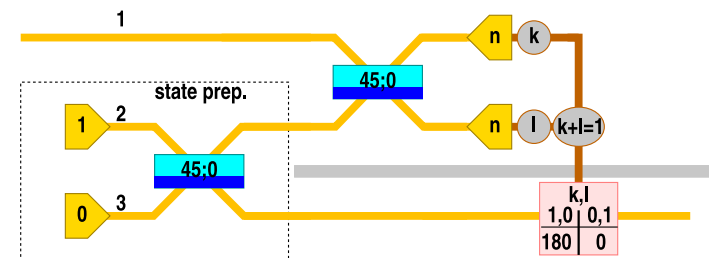
- Translation to photonic qubits.



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One Mode Teleportation

- Teleportation of one mode, success probability $1/2$:

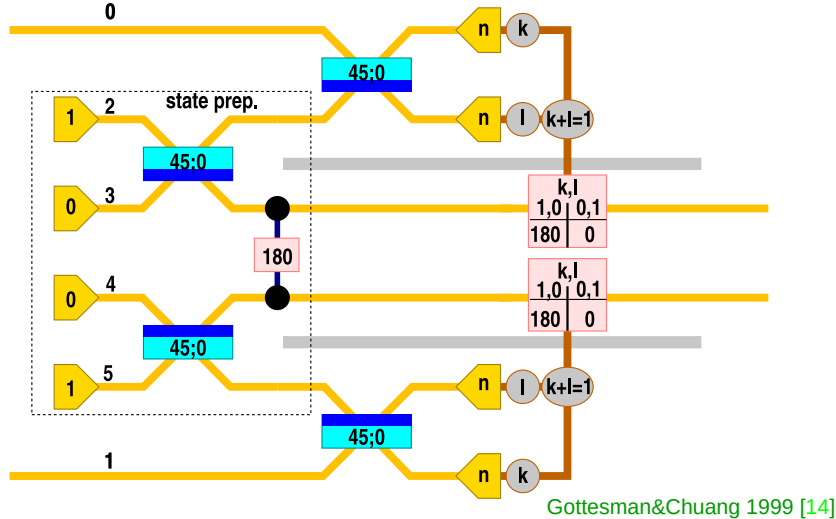


$$\begin{aligned}
 (\alpha|0\rangle_1 + \beta|1\rangle_1) &\longrightarrow \\
 (\alpha|0\rangle_1 + \beta|1\rangle_1) \frac{1}{\sqrt{2}} (|10\rangle_{23} + |01\rangle_{23}) &= \frac{1}{\sqrt{2}} (\alpha|010\rangle_{123} + \beta|101\rangle_{123}) \\
 &+ (0 \text{ or } 2 \text{ photons in modes } 1, 2) \longrightarrow \begin{cases} \frac{1}{2} (-\alpha|0\rangle_3 - \beta|1\rangle_3) \\ \text{or} \\ \frac{1}{2} (+\alpha|0\rangle_3 + \beta|1\rangle_3) \end{cases}
 \end{aligned}$$

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CS by Teleportation

- Implement CS using teleportation, probability of success $1/4$:



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Schemes for Improving the Success Probability

- Teleportation with $2n$ ancillas.
 - Generalization of one-mode teleportation.
 - State preparation complexity: Not efficient.
- With Z -measurement error-detecting codes.
 - Rely on failures of CS being unintentional measurement.
 - Logical qubits encoded in n photonic qubits.
 - Efficient.
- Using “linked photon circuits”.
 - Uses one photon per qubit.
 - Effects of CS failures are localized and enabling reconstruction.
 - More efficient.
- Using “cluster states”.
 - Uses photonic qubits.
 - Effects of CS failures are localized and enabling reconstruction.
 - Further efficiency improvements.

Yoran & Reznik 2003 [12]

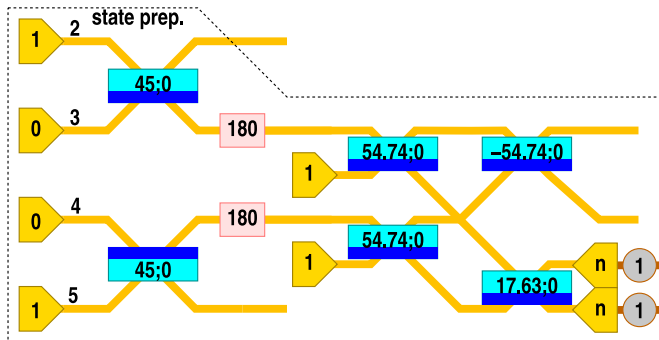
Nielsen 2004 [13]

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State Preparation for CS

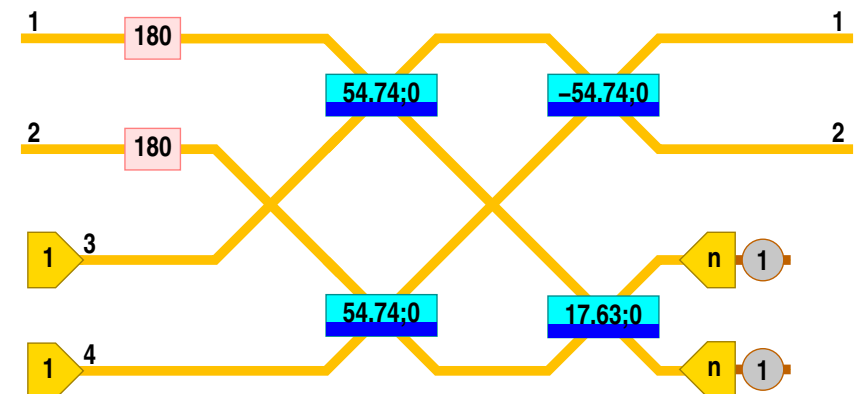
- A source of entangled pairs suffices.



- Or use: Two-mode non-linear sign shift with $\text{prob}_{\text{succ}} = 1/13.5$.

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Zooming in on eLOQC

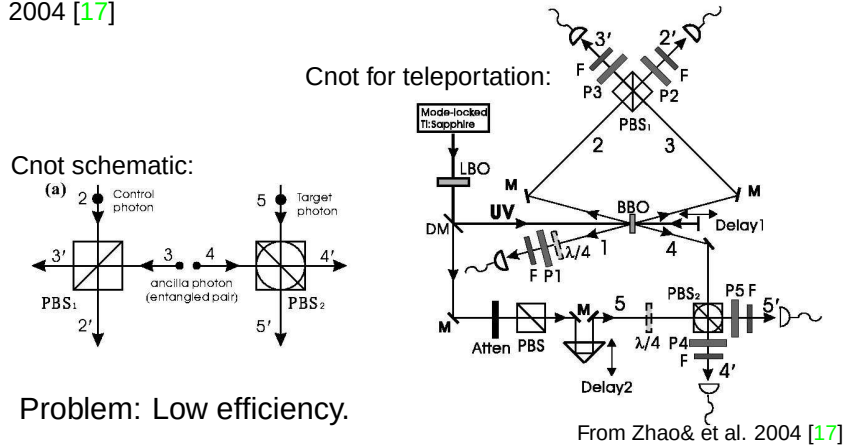


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Status of Experimental Efforts

- Current experiments use down-converted photons.
 - Australia: E.g. O'Brien&Pryde&White&Ralph&Branning 2003 [15]
 - US: E.g. Pittman&Jacobs&Franson 2004 [16]
 - Europe/China: E.g. Zhao&Zhang&Chen&Zhang&Du&Yang&Pan 2004 [17]



Problem: Low efficiency.

From Zhao& et al. 2004 [17]

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Challenges and Device Requirements

- Logical requirements.
 - Logical teleportation: $\ll 30\%$ detected, $\ll 10\%$ undetected error.
 - State preparation: $< 100\%$ detected, $\ll 5\%$ undetected error per qubit.
- Device requirement guesses.

	Probably ok?	I'll work harder?
Single photon source:	$> 99.9\%$	$> 90\%$
0, 1, 2-photon counter:	$> 99.9\%$	$> 90\%$
Photon loss (≈ 10 devices):	$\ll 1\%$	$< 10\%$
Beamsplitter (δ trans):	$\ll .1\%$	$< 1\%$
Mode matching (overlap ²):	$\ll .1\%$	$< 1\%$

- Further challenges.
 - Feed-forward: Time delay for classical decisions = τ_c .
 - Ph. counter: Detection time = τ_p .
 - Storage: Time to unacceptable loss = τ_s .
 - Switches: Low loss/fast.

$$\left. \begin{array}{l} \tau_c + \tau_p \ll \tau_s \\ \text{Back to: Guide} \end{array} \right\}$$

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