

Computer Simulations of Pedestrian Dynamics and Trail Formation

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Summary:

A simulation model for the dynamic behaviour of pedestrian crowds is mathematically formulated in terms of a social force model, that means, pedestrians behave in a way as if they would be subject to an acceleration force and to repulsive forces describing the reaction to borders and other pedestrians. The computational simulations presented yield many realistic results that can be compared with video films of pedestrian crowds. Especially, they show the self-organization of collective behavioural patterns.

By assuming that pedestrians tend to choose routes that are frequently taken the above model can be extended to an active walker model of trail formation. The topological structure of the evolving trail network will depend on the disadvantage of building new trails and the durability of existing trails. Computer simulations of trail formation indicate to be a valuable tool for designing systems of ways which satisfy the needs of pedestrians best. An example is given for a non-directed trail network.

Key-Words:

Social force model, pedestrian dynamics, self-organization of behavioural patterns, active walker model, trail formation

Introduction

During the last two decades the investigation of dynamic pedestrian behaviour has found notable interest among scientists from various disciplines for several reasons: First, architects and urban planners need powerful tools for designing pedestrian areas, subway or railroad stations, entrance halls, shopping malls, escape routes, etc. Second, there are some theoretical challenges like the description of interaction effects (that can lead to jams, blockages) and of influences of the geometry of pedestrian facilities. The observation of striking analogies with gases and fluids is of particular interest for physicists. Third, pedestrian models can be experimentally tested, since all model quantities like places and velocities of pedestrians are easily measurable. Empirical data material of flow measurements and film material already exist. Fourth, pedestrian models can give valuable hints for the development of more general or other behavioural models. For example, there exist analogies with models of opinion formation [1,2].

The historical modelling approaches to pedestrian dynamics were quite different. Traffic engineers usually try to fit simple *regression models* to flow measurements [3]. However, since these models do not describe interaction effects in an adequate way, these flow measurements and regression analyses have to be performed for every special situation. In particular, it is not possible to exactly predict the pedestrian flows in pedestrian areas or buildings with an extraordinary architecture during the planning phase. There have also been suggested some *queuing models* [4,5]. However, these are only suitable for certain kinds of questions, since they do not explicitly take into account the effects of the concrete geometry of pedestrian facilities. Furthermore, BORGERS and TIMMERMANS [6] have developed a model for the *route choice behaviour* of pedestrians in dependence of their demands, the city entry points, and the store locations. However, this approach does not model the effects of pedestrian interactions, in spite of the fact that pedestrians will take detours if their preferred way is crowded. In the 1970s, HENDERSON suggested that pedestrian crowds can be described by the NAVIER-STOKES equations of *fluid-dynamics* [7]. However, his approach implicitly assumes energy and momentum to be collisional invariants which is obviously not the case for pedestrian interactions. Therefore, in the 1990s a fluid-dynamic pedestrian model has been derived on the basis of a pedestrian specific *gaskinetic* (BOLTZMANN-like) model [8,1]. This model starts from observations of individual pedestrian behaviour and takes into account the intentions, desired velocities and pair interactions of the considered pedestrians.

Apart from analytical investigations, for practical applications a direct simulation of individual pedestrian behaviour is favourable, since a numerical solution of the fluid-dynamic equations is very difficult. As a consequence, current research focuses on the microsimulation of pedestrian crowds. In this connection, a *social force model* of pedestrian dynamics has been suggested [9,10] which is related to *molecular dynamics* [11]. A simple forerunner of this kind of model has been proposed by GIPPS and MARKSJÖ [12].

The social force model can be extended to an *active walker model* [13–16] by inclusion of a *trail formation mechanism*, which has originally been suggested for the description of trunk trail formation by ants [16]. One interesting application of this active walker model is the construction of way systems that are optimal compromises between *minimal way systems* (that necessitate many detours) and *direct way systems* (that are very expensive and need too much space) [17].

The social force model of pedestrian dynamics

Pedestrians are *used* to the situations they are normally confronted with. Their behaviour is determined by their experience which reaction to a certain stimulus (situation) will be the *best*. Therefore, their reactions are usually rather ‘*automatic*’ and well predictable. The corresponding behavioural rules can be put into an *equation of motion*. According to this equation, changes of the *actual velocity* $\vec{v}_\alpha(t)$ of a pedestrian α are given by a vectorial quantity $\vec{f}_\alpha(t)$:

$$\frac{d\vec{v}_\alpha}{dt} = \vec{f}_\alpha(t) + \text{fluctuations}. \quad (1)$$

$\vec{f}_\alpha(t)$ can be interpreted as *social force* that describes the influence of environment and other pedestrians on the individual behaviour. However, the social force is not *exerted* on a pedestrian. It rather describes the concrete *motivation to act*. The *fluctuation term* takes into account random variations of the behaviour, which can arise, for example, in situations where two or more behavioural alternatives are equivalent, or by accidental or deliberate deviations from the usual rules of motion.

There are some major differences between social forces and forces in physics. First, for social forces the NEWTONian law *actio = reactio* does not hold. Second, energy and momentum are *no collisional invariants* which implies that there is no energy or momentum conservation. Third, pedestrians (or, more general, individuals) are *active systems*, that produce forces and perform changes themselves. Fourth, instead of by *momentum transfer* via virtual particles the effect of social forces comes about by *information exchange* via complex mental, psychological and physical processes.

In the following we will specify the social force model of pedestrian motion:

1. A pedestrian α wants to walk into a *desired direction* \vec{e}_α (the direction of his/her next destination) with a certain *desired speed* v_α^0 . The desired speeds of pedestrians are almost normally distributed:

$$P(v^0) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(v^0 - \langle v^0 \rangle)^2 / (2\sigma^2)}. \quad (2)$$

A deviation of the *actual velocity* \vec{v}_α from the *desired velocity* $\vec{v}_\alpha^0 := v_\alpha^0 \vec{e}_\alpha$ leads to a tendency \vec{f}_α^0 to approach \vec{v}_α^0 again within a certain *relaxation time* τ_α . This can be described by an *acceleration term* of the form

$$\vec{f}_\alpha^0(\vec{v}_\alpha, v_\alpha^0 \vec{e}_\alpha) := \frac{1}{\tau_\alpha} (v_\alpha^0 \vec{e}_\alpha - \vec{v}_\alpha). \quad (3)$$

\vec{f}_α^0 is a measure for the ‘*pressure of time*’, that means the *motivation to get ahead*.

2. Pedestrians keep a certain distance to *borders* (of buildings, walls, streets, obstacles, etc.). This effect $\vec{f}_{\alpha B}$ can be described by a repulsive, monotonic decreasing potential V_B :

$$\vec{f}_{\alpha B}(\vec{r}_\alpha - \vec{r}_B^\alpha) = -\nabla_{\vec{r}_\alpha} V_B(\|\vec{r}_\alpha - \vec{r}_B^\alpha\|). \quad (4)$$

Here, \vec{r}_α is the actual *location* of pedestrian α . In addition, \vec{r}_B^α denotes the location of the piece of the border that is nearest to location \vec{r}_α .

3. The motion of a pedestrian α is influenced by other pedestrians β . These interactions cause him/her to perform *avoidance manoeuvres* or to slow down in order to keep a situation dependent distance to other pedestrians. A quite realistic description of pedestrian interactions results from the assumption that each pedestrian respects the ‘*privat spheres*’ of other pedestrians β . These *territorial effects* $\vec{f}_{\alpha\beta}$ can be modelled by *repulsive potentials* $V_\beta(b)$:

$$\vec{f}_{\alpha\beta}(\vec{r}_\alpha - \vec{r}_\beta) = -\nabla_{\vec{r}_\alpha} V_\beta[b(\vec{r}_\alpha - \vec{r}_\beta)] . \quad (5)$$

The sum over the repulsive potentials V_β defines the *interaction potential* which influences the behaviour of each pedestrian:

$$V_{\text{int}}(\vec{r}, t) := \sum_{\beta} V_\beta\{b[\vec{r} - \vec{r}_\beta(t)]\} . \quad (6)$$

For $V_\beta(b)$ we will assume a monotonic decreasing function in b with equipotential lines having the form of an ellipse that is directed into the direction of motion (see Fig. 1). The reason for this is that a pedestrian requires space for the next step, which is taken into account by other pedestrians. b denotes the *semi-minor axis* of the ellipse and is given by

$$2b = \sqrt{(\|\vec{r}_\alpha - \vec{r}_\beta\| + \|\vec{r}_\alpha - \vec{r}_\beta - v_\beta \Delta t \vec{e}_\beta\|)^2 - (v_\beta \Delta t)^2} . \quad (7)$$

$s_\beta := v_\beta \Delta t$ is about the *step width* of pedestrian β .



Fig. 1: Left: Decrease of the repulsive interaction potential $V_\beta(b)$ with b . Right: Elliptical equipotential lines of $V_\beta[b(\vec{r} - \vec{r}_\beta)]$ for different values of b .

Since all the effects 1 to 3 influence a pedestrian's decision at the *same* moment, we shall assume that their total effect is given by the *sum* of all effects like this is the case for forces. The *social force (total motivation)* \vec{f}_α is, therefore, given by

$$\begin{aligned} \vec{f}_\alpha(t) &:= \vec{f}_\alpha^0(\vec{v}_\alpha, v_\alpha^0 \vec{e}_\alpha) + \vec{f}_{\alpha B}(\vec{r}_\alpha - \vec{r}_B^\alpha) + \sum_{\beta(\neq \alpha)} \vec{f}_{\alpha\beta}(\vec{r}_\alpha - \vec{r}_\beta) \\ &= \frac{1}{\tau_\alpha} (v_\alpha^0 \vec{e}_\alpha - \vec{v}_\alpha) - \nabla_{\vec{r}_\alpha} [V_B(\|\vec{r}_\alpha - \vec{r}_B^\alpha\|) + V_{\text{int}}(\vec{r}_\alpha, t)] . \end{aligned} \quad (8)$$

According to (1), $\vec{f}_\alpha(t)$ determines the temporal change of the actual velocity $\vec{v}_\alpha(t)$. Together with the equation

$$\frac{d\vec{r}_\alpha}{dt} := v_\alpha(t) , \quad (9)$$

(1) and (8) characterize the *social force model* of pedestrian motion. The corresponding model of pedestrian *crowds* has the form of *nonlinearly coupled stochastic differential equations*.

The social force model (1), (8), (9) of pedestrian dynamics has been simulated on a computer for a large number of interacting pedestrians confronted with different situations. Despite the

fact that the proposed model is very simple it describes a lot of observed phenomena very realistically. For example, under certain conditions the *emergence* of new spatio-temporal patterns of collective behaviour can be observed:

1. the development of lanes (groups) consisting of pedestrians walking into the *same* direction (see Fig. 2),
2. oscillatory changes of the walking direction at narrow passages (for example, doors) (see Fig. 3),
3. the spontaneous formation of roundabout traffic at intersections (see Fig. 4).

This formation of new spatio-temporal patterns is called ‘*self-organization*’ since it is not caused by the special *initial* or *boundary conditions*, but by the *non-linear interactions* of pedestrians. Furthermore, the self-organization phenomena mentioned above are not the effect of strategical considerations, because the pedestrians were assumed to behave in a rather ‘automatic’ way.

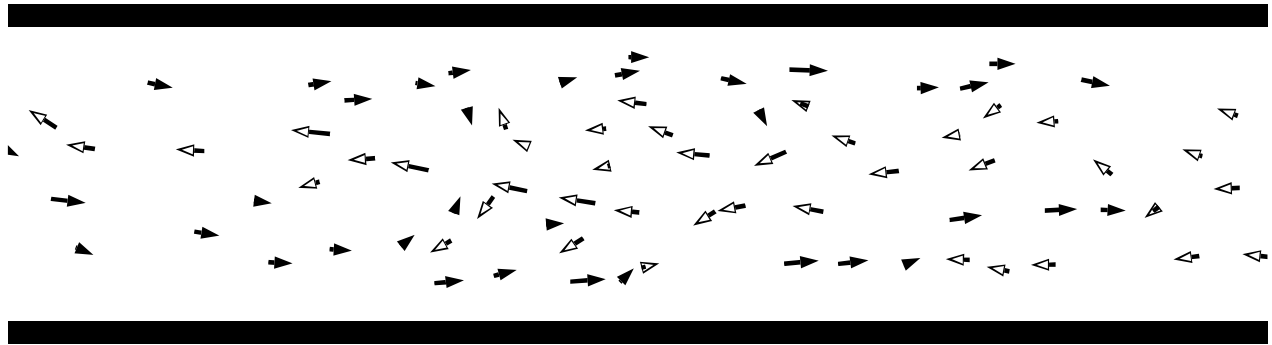


Fig. 2: Above a critical pedestrian density one finds the formation of lanes consisting of pedestrians with an uniform walking direction. Here, the computational result shows three lanes. Black arrows represent pedestrians who desire to walk from left to right, white arrows pedestrians who want to move into the opposite direction. The lengths of the arrows indicate the speeds of the corresponding pedestrians.

Trail formation of pedestrians

In the following, we will focus our interest on the self-organization of *systems of ways*, which has been described recently by means of an *active walker model* [16]. The term ‘*active walker*’ [13–16] is derived from ‘*random walker*’ which means a particle that moves randomly in space due to the effect of fluctuations. An active walker is *also* subject to fluctuations, but is additionally able to perform certain actions:

1. An active walker can change his/her/its environment by setting *markings*.
2. In addition, an active walker is able to *read* these markings which influence his/her/its behaviour in a specific way.

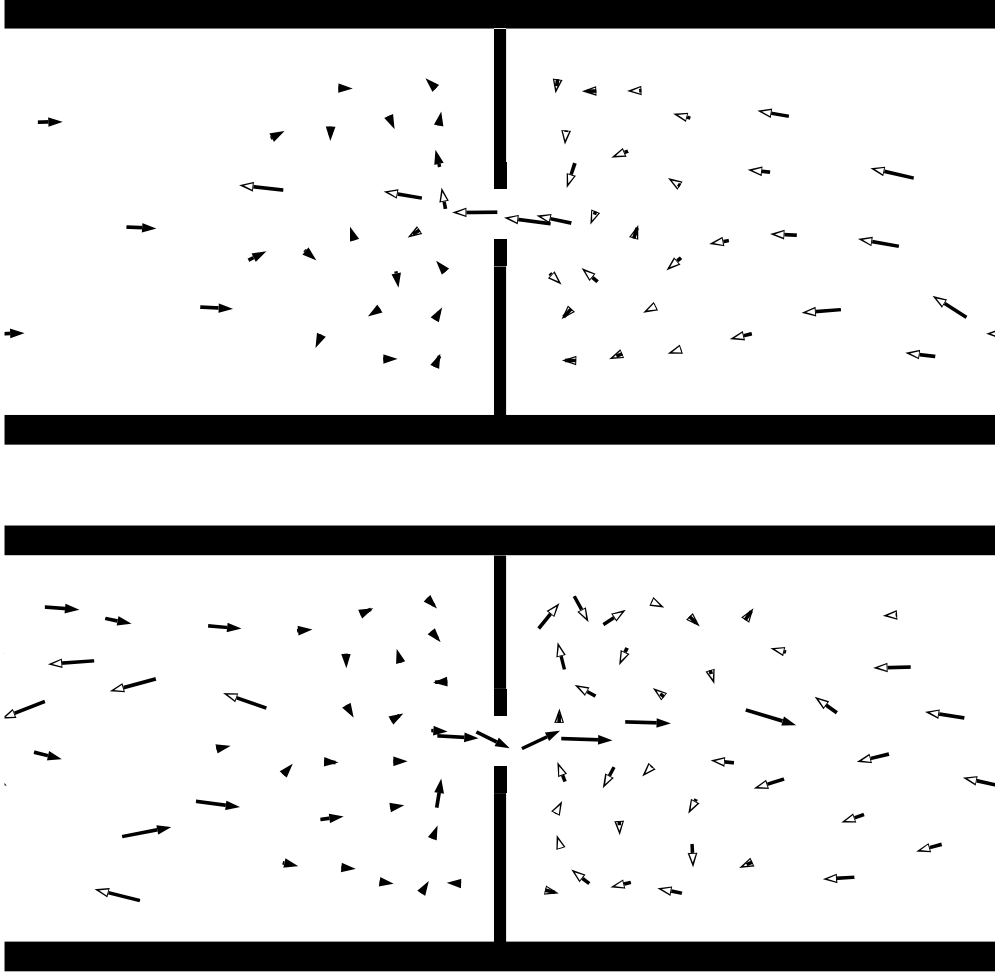


Fig. 3: Different moments of two pedestrian groups that try to pass a narrow door into opposite directions. If one pedestrian has been able to pass a narrow door, other pedestrians with the same desired walking direction can follow easily whereas pedestrians with an opposite desired direction of motion have to wait (above). After some time the pedestrian stream is stopped by the pressure of the opposing group, and the door is subsequently captured by pedestrians who intend to pass the door into the opposite direction (below). The change of the passing direction may occur several times.

That means, active walkers *interact indirectly* with each other via environmental changes (markings). The markings have their own properties which depend on their special type. For example, they may decay with time or diffuse spatially.

The pedestrian dynamics discussed above can be expanded with a trail formation mechanism by assuming that each pedestrian produces a *trail* consisting of *footprints* (which play the role of markings) [15,16]. These footprints will (exponentially) decay in the course of time with a rate $1/T$ where T means the *durability* of the markings. The decay rate $1/T$ should not be too large since then the trails will almost have vanished before another pedestrian intends to take a similar route. Introducing a *trail potential* $V_{\text{tr}}(\vec{r}, t)$ which describes the additional effect $\vec{f}_{\text{tr}}(\vec{r}, t) := -\nabla_{\vec{r}} V_{\text{tr}}(\vec{r}, t)$ of the markings on pedestrian motion, the temporal change of $V_{\text{tr}}(\vec{r}, t)$ is given by a *decay term* and terms $Q_{\alpha}(\vec{r}, t)$ which reflect the *production* of new footprints by the pedestrians:

$$\frac{dV_{\text{tr}}(\vec{r}, t)}{dt} = -\frac{1}{T} V_{\text{tr}}(\vec{r}, t) + \sum_{\beta} Q_{\beta}(\vec{r}, t). \quad (10)$$

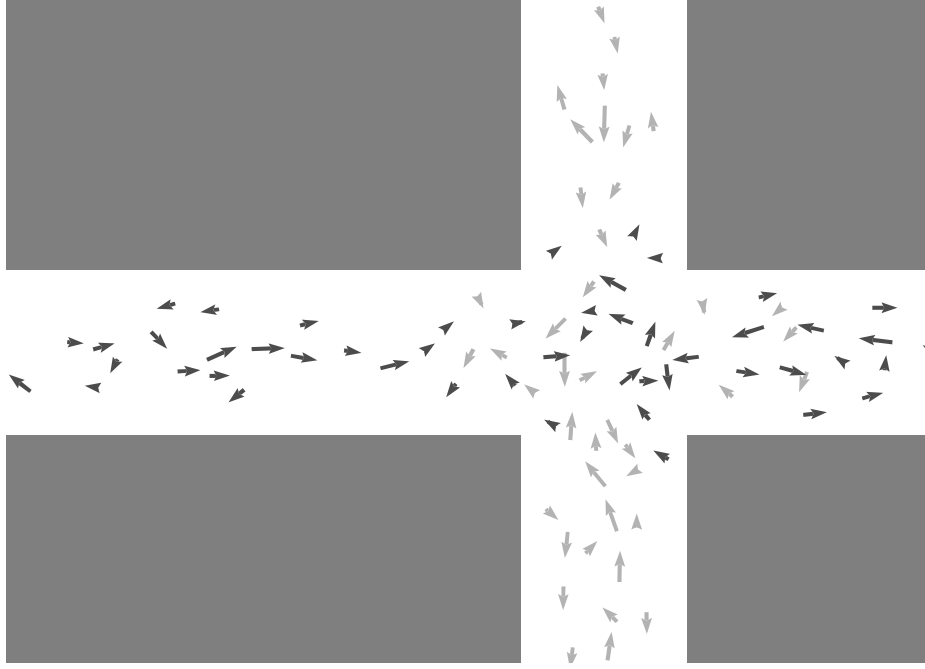


Fig. 4: At crowded intersections there develops roundabout traffic for certain time periods. The related rotation direction changes from time to time. White arrows represent pedestrians who desire to walk in vertical direction, black arrows pedestrians who want to move in horizontal direction.

Integration of (10) leads to

$$V_{\text{tr}}(\vec{r}, t) = \int_{t_0}^t dt' Q(\vec{r}, t') \exp\left(-\frac{t-t'}{T}\right) \quad \text{with} \quad Q(\vec{r}, t) := \sum_{\beta} Q_{\beta}(\vec{r}, t). \quad (11)$$

The *production terms* $Q_{\alpha}(\vec{r}, t)$ are modelled by *attractive potentials*, since it should be easier (more convenient) for pedestrians to take already existing trails than to clear new ways. For reasons of simplicity we will assume that $Q_{\alpha}(\vec{r}, t)$ has the same functional form as the repulsive potential $V_{\alpha}\{b[\vec{r} - \vec{r}_{\alpha}(t)]\}$, but with an opposite sign. If the parameter q reflects the strength of the attraction of new markings, we then have

$$Q_{\alpha}(\vec{r}, t) = -qV_{\alpha}\{b[\vec{r} - \vec{r}_{\alpha}(t)]\} \quad \text{and} \quad Q(\vec{r}, t) = -qV_{\text{int}}(\vec{r}, t). \quad (12)$$

q can be interpreted as the *advantage of using existing trails*. If q is too small, no trail formation will be observed. On the other hand, if q is too large, the attraction effect of trails will destroy the repulsive effect of pedestrian interactions which would lead to collisions of pedestrians. A trail formation, of course, is observed only if the trails are reinforced by regular usage.

In our simulations we use a triangular grid for the two-dimensional space and a discretization of time. The related discretization of (11) reads then:

$$V_{\text{tr}}(\vec{r}, t) = \Delta t \sum_{n=1}^{\infty} Q(\vec{r}, t - n \Delta t) \exp\left(-\frac{n \Delta t}{T}\right). \quad (13)$$

Moreover, the equation of motion for a pedestrian α obtains the form

$$\frac{\vec{v}_{\alpha}(t + \Delta t) - \vec{v}_{\alpha}(t)}{\Delta t} = \frac{1}{\tau_{\alpha}}[v_{\alpha}^0 \vec{e}_{\alpha} - \vec{v}_{\alpha}(t)] - \nabla_{\vec{r}_{\alpha}} V_{\text{tot}}(\vec{r}_{\alpha}, t) + \text{fluctuations} \quad (14)$$

where we have introduced the *total potential*

$$V_{\text{tot}}(\vec{r}, t) = V_B(\|\vec{r} - \vec{r}_B\|) + V_{\text{int}}(\vec{r}, t) + V_{\text{tr}}(\vec{r}, t). \quad (15)$$

Computer simulations of the above described trail formation model are expected to be a useful tool for designing efficient systems of ways that satisfy the pedestrians' requirements. For this purpose one has to specify the advantage q of using existing trails and to simulate the expected flows of pedestrians that enter the considered system at certain entry points with the intention to reach certain destinations. According to the above described model, after some time a way system will evolve which takes into account the pedestrians' route choice habits. For the simulation of real situations the corresponding results may serve as suitable planning guidelines for architects or urban planners.

Formation of non-directed trail systems

The social force model of pedestrian motion described above assumes of each pedestrian certain intentions like a desired direction, desired speed, and desired distance from obstacles or other pedestrians. Whereas these assumptions allow us to describe the interactions of 'real' pedestrians, we now want to focus on the *basic* features of the process of trail formation itself. Therefore, we drop some of the assumptions above and simplify the model. The pedestrians are assumed to be *simple* walkers which only make *local* decisions about the direction of their next step. In particular, these walkers have *no memory* about the way they have gone or about certain destinations.

During their walk, the walkers leave markings ('footprints') again. If we neglect the repulsive interactions between the walkers and assume no obstacles in the system, the only force on the walkers is given by the trail potential $V_{\text{tr}}(\vec{r}, t)$. The equation of motion of walker α is now given by equation (9) and

$$\frac{d\vec{v}_\alpha}{dt} = -\gamma\vec{v}_\alpha - \nabla_{\vec{r}_\alpha} V_{\text{tr}}(\vec{r}_\alpha, t) + \sqrt{2\gamma^2 D} \xi_\alpha(t) \quad (16)$$

where we have added a dissipative '*friction term*' $-\gamma\vec{v}_\alpha$ which describes a loss of energy. γ is the *friction coefficient* of the walkers. The last term of (16) specifies the fluctuation term of equation (1) as a *white random force (white noise)*, where the strength of the fluctuations is related to the *diffusion coefficient* D . It is well known that equations (9) and (16) can, in the SMOLUCHOWSKI limit [18], be reduced to

$$\frac{d\vec{r}_\alpha}{dt} = -\frac{1}{\gamma} \nabla_{\vec{r}_\alpha} V_{\text{tr}}(\vec{r}_\alpha, t) + \sqrt{2D} \xi_\alpha(t) \quad (17)$$

which corresponds to $d\vec{v}_\alpha/dt \approx 0$. The first part of (17) describes the fact that the walkers tend to follow the *gradient* of the markings (that means, to move into the direction of the *strongest* marking in their neighbourhood), whereas the second part represents the fluctuations that keep them moving otherwise with a certain probability. As long as the gradient of $V_{\text{tr}}(\vec{r}, t)$ is small and the fluctuations are large, the walkers behave nearly as random walkers, but the action of the walkers (the production of markings) can lead to a *supercritical value* of the attractive trail potential which causes a *spatially restricted* motion [15].

Equation (12) for the production of markings implies that the footprints have an attraction potential with a certain spatial extension, expressed by the parameter b . This means a trail

becomes already attractive in a certain distance from the trail itself. If we assume instead that the attraction potential is sharply peaked around the markings, the production term can be simplified by means of δ -functions $\delta[\vec{r} - \vec{r}_\alpha(t)]$ at the actual positions \vec{r}_α of the walkers:

$$Q(\vec{r}, t) := -q \sum_{\beta} \delta[(\vec{r} - \vec{r}_\beta(t))]. \quad (18)$$

Furthermore, we suppose that the walkers are able to see a marked trail only in a certain range of space (a certain angle of view in the direction of their movement). That means they percept their environment similar to real pedestrians rather than to physical particles which ‘feel’ a gradient in every direction. If the walkers find markings within the perceived range of space, they can choose the direction of their next step towards the strongest marking, but, due to the fluctuations, with a certain probability they ‘ignore’ the markings and make a random decision on their next step. Provided that they follow the markings, the footprints are reinforced.

As a result, a trail system will evolve (see Fig. 5) which consists of a number of major and minor trails. The frequency of use is coded with a grey scale. This trail system is non-directed, it does not connect any points of destination. Since the simulated surface is only *partially* covered with markings, the walkers really use the trails during their movement instead of moving around anywhere.

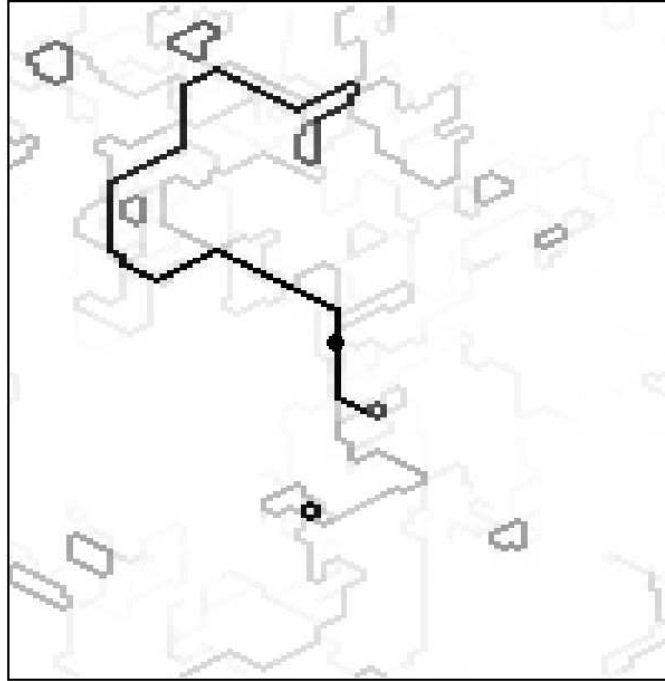


Fig. 5: Non-directed trail system originated by 100 active walkers after 5000 time steps. (The starting point was in the middle of the triangular lattice of size 100x100.)

The observed trail system is a *non-planned structure*. It emerges from the *local interaction* of the walkers with the surface, which in turn changes the movement of the walkers. There is an interplay between the reinforcement of trails and the *selection* of trails that are less used. The resulting structure finally consists of only those trails which could be maintained by the walkers. All other trails have vanished after a certain time.

As indicated by (17) the density of the trail system depends on the relation between the attractive and the dissipative effects. If the diffusion constant D is small, only a few trails

will evolve. The density of trails increases with D , and above a critical value D_{crit} for D , a sharply distinguished trail system is no longer observed (every walker creates its own trail, then) [19].

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