Lecture Note

Application of Fuzzy Logic

Akira Imada
Brest State Technical University

Last modified on 26 November 2015

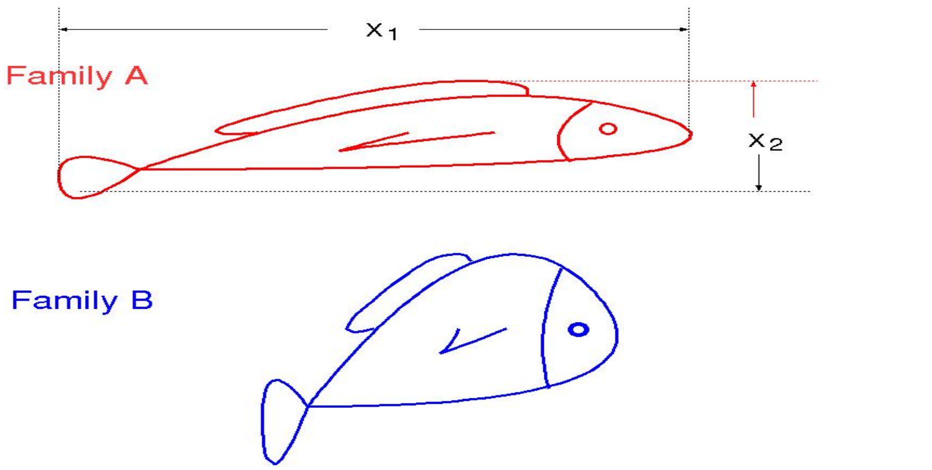
I. Fuzzy Basic Arithmetics

II. Fuzzy Controller

III. Fuzzy Classification

3. Takagi Sugeno Formula

An example of classification - 3 families of fish



3 rules to classify as an example

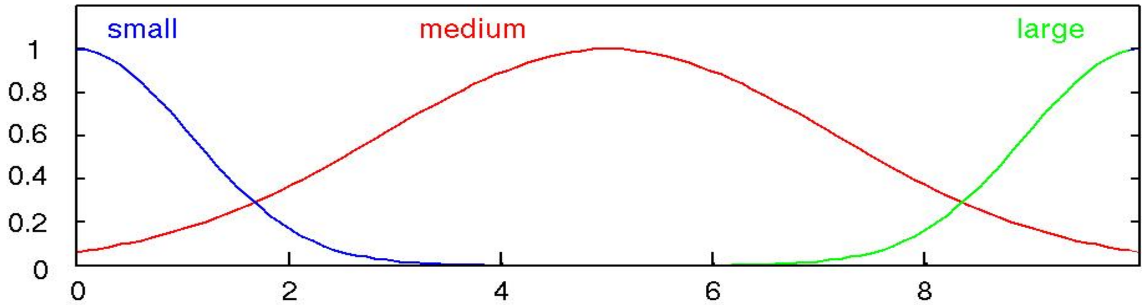
R_1: IF X_1 = medium AND X_2 = small THEN A

R_2: IF X_1 = small AND X_2 = medium THEN B

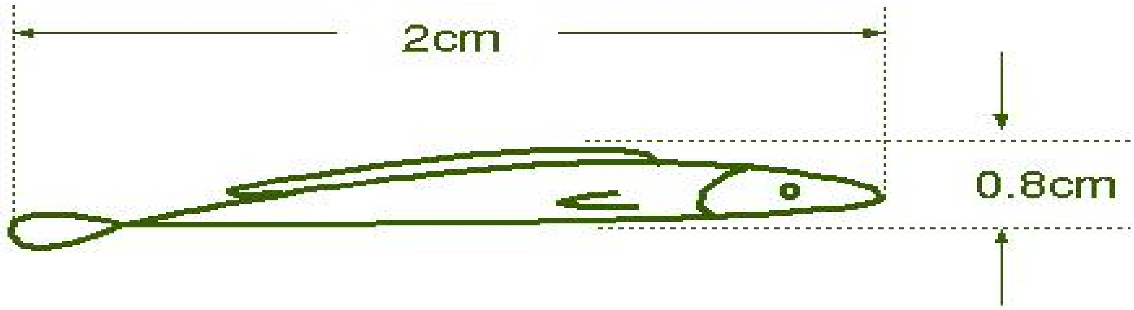
R_3: IF X_1 = large AND X_2 = small THEN C

Memership function for the size of two parts

$$\mu(x) = \exp\{-\frac{(x-avg)^2}{\sigma^2}\}$$



Qestion: Which family is this new fish?



1. Singleton Consequence

R_k: If x_1 is A_1^k, and x_2 is A_2^k and $\cdots$ and x_N is A_N^k then y is g^k.

Takagi-Sugeno rules: Estimation of a single input

Estimation of y for an input $\mathbf{x} = (x_1, x_2, \cdots, x_N)$

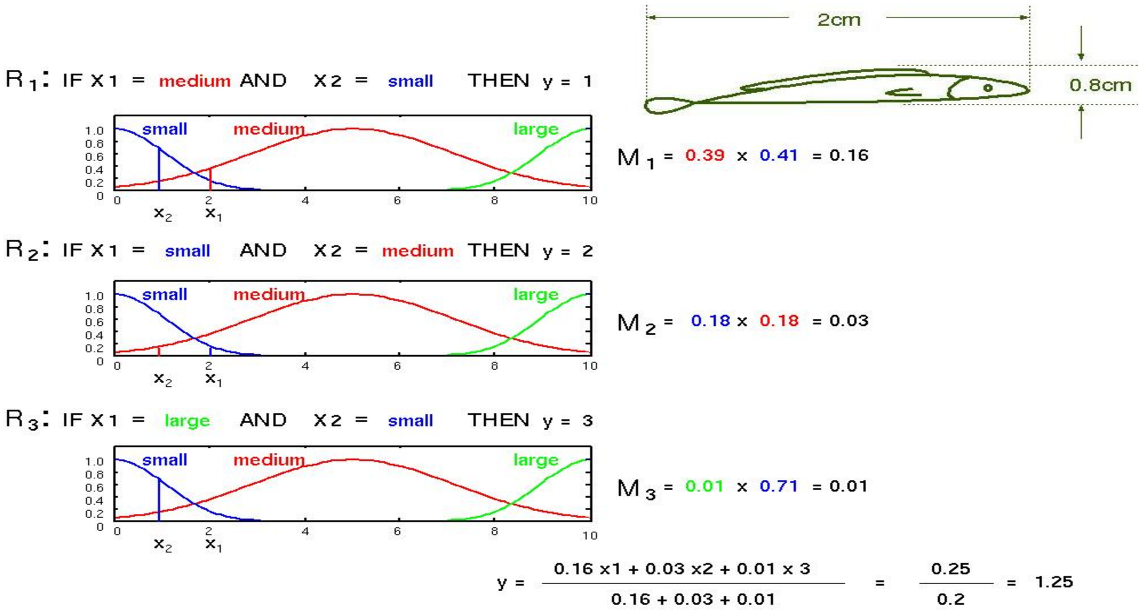
$$y_j = \frac{\sum_{k=1}^{H}(M_k(\mathbf{x}) \cdot g_k)}{\sum_{k=1}^{H} M_k(\mathbf{x})}$$

where

$$M_k(\mathbf{x}) = \prod_{i=1}^{N} \mu_{ik}(x_i)$$

where μ_{ik} is i-th attribute of k-th rule

Three rules to classify



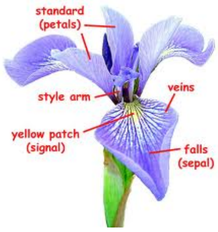
A benchmark – Iris database

Iris flower dataset (taken from University of California Urvine Machine Learning Repository) consists of three species of iris flower

setosa, versicolor and *virginica.*

Each sample represents four attributes of the iris flower

sepal-length, sepal-width, petal-length, and *petal-width.*



Iris Flower Database

Setosa				Versicolor				Virginica			
x_1	x_2	x_3	x_4	x_1	x_2	x_3	x_4	x_1	x_2	x_3	x_4
0.65	0.80	0.20	0.08	0.89	0.73	0.68	0.56	0.80	0.75	0.87	1.00
0.62	0.68	0.20	0.08	0.81	0.73	0.65	0.60	0.73	0.61	0.74	0.76
0.59	0.73	0.19	0.08	0.87	0.70	0.71	0.60	0.90	0.68	0.86	0.84
0.58	0.70	0.22	0.08	0.70	0.52	0.58	0.52	0.80	0.66	0.81	0.72
0.63	0.82	0.20	0.08	0.82	0.64	0.67	0.60	0.82	0.68	0.84	0.88
0.68	0.89	0.25	0.16	0.72	0.64	0.65	0.52	0.96	0.68	0.96	0.84
0.58	0.77	0.20	0.12	0.80	0.75	0.68	0.64	0.62	0.57	0.65	0.68
0.63	0.77	0.22	0.08	0.62	0.55	0.48	0.40	0.92	0.66	0.91	0.72

(to be cont'd to the next page)

Setosa				Versicolor				Virginica			
x_1	x_2	x_3	x_4	x_1	x_2	x_3	x_4	x_1	x_2	x_3	x_4
0.56	0.66	0.20	0.08	0.84	0.66	0.67	0.52	0.85	0.57	0.84	0.72
0.62	0.70	0.22	0.04	0.66	0.61	0.57	0.56	0.91	0.82	0.88	1.00
0.68	0.84	0.22	0.08	0.63	0.45	0.51	0.40	0.82	0.73	0.74	0.80
0.61	0.77	0.23	0.08	0.75	0.68	0.61	0.60	0.81	0.61	0.77	0.76
0.61	0.68	0.20	0.04	0.76	0.50	0.58	0.40	0.86	0.68	0.80	0.84
0.54	0.68	0.16	0.04	0.77	0.66	0.68	0.56	0.72	0.57	0.72	0.80
0.73	0.91	0.17	0.08	0.71	0.66	0.52	0.52	0.73	0.64	0.74	0.96
0.72	1.00	0.22	0.16	0.85	0.70	0.64	0.56	0.81	0.73	0.77	0.92
0.68	0.89	0.19	0.16	0.71	0.68	0.65	0.60	0.82	0.68	0.80	0.72
0.65	0.80	0.20	0.12	0.73	0.61	0.59	0.40	0.97	0.86	0.97	0.88
0.72	0.86	0.25	0.12	0.78	0.50	0.65	0.60	0.97	0.59	1.00	0.92
0.65	0.86	0.22	0.12	0.71	0.57	0.57	0.44	0.76	0.50	0.72	0.60
0.68	0.77	0.25	0.08	0.75	0.73	0.70	0.72	0.87	0.73	0.83	0.92
0.65	0.84	0.22	0.16	0.77	0.64	0.58	0.52	0.71	0.64	0.71	0.80
0.58	0.82	0.14	0.08	0.80	0.57	0.71	0.60	0.97	0.64	0.97	0.80
0.65	0.75	0.25	0.20	0.77	0.64	0.68	0.48	0.80	0.61	0.71	0.72
0.61	0.77	0.28	0.08	0.81	0.66	0.62	0.52	0.85	0.75	0.83	0.84

Excercise

1. *Design 3 Gaussian membership function for SMAL, MEDIUM and LARGE.*
2. *Design 6 rules to classify iris flower.*
3. *Make codes whose inputs are x_1, x_2, x_3 and x_4, and output is A, B or C.*
4. *Evaluate your algorithm by using all of the 25 data above and total estimate by*

$$f = \frac{Number\ of\ successes}{75}.$$

Result

A, B or C	x_1	x_2	x_3	x_4	g	$\hat{y}$	OK or NOT
A	0.65	0.80	0.20	0.08			
A	0.62	0.68	0.20	0.08			
	......						
C	0.85	0.75	0.83	0.84			

2. Stochastic Consequence

R_k: If x_1 is A_1^k, and $\cdots$ and x_N is A_N^k
then
y_1 is g_1^k and $\cdots$ and y_N is g_N^k.

Result

A, B or C	x_1	x_2	x_3	x_4	y_1	y_2	y_3	y_4	$\hat{y}$	OK or NOT
A	0.65	0.80	0.20	0.08						
A	0.62	0.68	0.20	0.08						
...	......									
C	0.85	0.75	0.83	0.84						

$$y_i = \frac{M_1 \cdot g_i^1 + M_2 \cdot g_i^2 + \cdots + M_6 \cdot g_i^6}{M_1 + M_1 + \cdots + M_6}$$

3. Linear Regression Consequence

R_i: If x_1 is A_1^i and x_2 is A_2^i and $\cdots$ and x_n is A_n^i
then

$$y = a_1^i x_1 + a_2^i x_1 + \cdots + a_n^i x_n + b^i.$$

Result

	a_1	a_2	a_3	a_4
Rule 1				
Rule 2				
Rule 3				
Rule 4				
Rule 5				
Rule 6				

A, B or C	x_1 x_2 x_3 x_4	y	$\hat{y}$	OK or NOT
A	0.65 0.80 0.20 0.08			
A	0.62 0.68 0.20 0.08			
...	......			
C	0.85 0.75 0.83 0.84			

$$y = \frac{M_1 \cdot y_1 + M_2 \cdot y_2 + \cdots + M_6 \cdot y_6}{M_1 + M_1 + \cdots + M_6}$$

IV. Fuzzy Clustering

1. Fuzzy Relation

2. Combining two Fuzzy Relations

Max-Min Composition Formula

$$\mu_{R_{A\circ B}}(x,z) = \max\{\min_{y}\{\mu_{R_A}(x,y), \mu_{R_B}(y,z)\}\}$$

E.g.

$$\begin{gathered} \mu_R \circ \mu_R(2,3) \\ = \max\{\min(0.4,0.5), \min(0.5,0.8), \min(0.6,0)\} \\ = \max\{0.4, 0.5, 0\} \\ = 0.5 \end{gathered}$$

when

$$\begin{bmatrix} 0.1 & 0.2 & 0.3 \\ 0.4 & 0.5 & 0.6 \\ 0.7 & 0.8 & 0.9 \end{bmatrix} \circ \begin{bmatrix} 0.3 & 0.1 & 0.5 \\ 0.2 & 0.7 & 0.8 \\ 0.9 & 0.4 & 0 \end{bmatrix}$$

3. Clustering by Similarity

Algorithm 1 *1. Calculate a max-min similarity-relation* $R = [a_{ij}]$

2. *Set* $a_{ij} = 0$ *for all* $a_{ij} < \alpha$ *and* $i = j$
3. *Select* s *and* t *so that* $a_{ij} = \max\{a_{ij} | i < j \text{ and } i, j \in I\}$. *When tie, select one of these pairs at random*

   WHILE $a_{st} \neq 0$ *DO put* s *and* t *into the same cluster* $C = \{s, t\}$ *ELSE [4.]*
   ELSE all indices $\in I$ *into separated clusters and STOP*

4. *Choose* $u \in I \setminus C$ *so that*

$$\sum_{i \in C} a_{iu} = \max_{j \in I \setminus C} \{\sum_{i \in C} a_{ij} | a_{ij} \neq 0\}$$

   When a tie, select one such u *at random.*

   WHILE such a u *exists, put* u *into* $C = \{s, t, u\}$ *and REPEAT [4.]*

5. *Let* $I = I \setminus C$ *and GOTO [3.]*

Repeat until no change, e.g.,

Starting with

$$\begin{bmatrix} 1 & 0.2 & 0.3 \\ 0.4 & 1 & 0.6 \\ 0.7 & 0.8 & 1 \end{bmatrix} \circ \begin{bmatrix} 1 & 0.2 & 0.3 \\ 0.4 & 1 & 0.6 \\ 0.7 & 0.8 & 1 \end{bmatrix}$$

repeat composition until no change

$$\begin{bmatrix} 1 & 0.2 & 0.3 \\ 0.4 & 1 & 0.6 \\ 0.7 & 0.8 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0.3 & 0.3 \\ 0.6 & 1 & 0.6 \\ 0.7 & 0.8 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0.3 & 0.3 \\ 0.8 & 1 & 0.6 \\ 0.8 & 0.8 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0.3 & 0.3 \\ 0.8 & 1 & 0.6 \\ 0.8 & 0.8 & 1 \end{bmatrix}$$

Example 1 *Starting from the following* 10×10 *proximity-relation* $R^{(0)}$*, let's apply the the algorithm above. Assume now* $\alpha = 0.55$*.*

$$R^{(0)} = \begin{bmatrix} 1 & .7 & .5 & .8 & .6 & .6 & .5 & .9 & .4 & .5 \\ .7 & 1 & .3 & .6 & .7 & .9 & .4 & .8 & .6 & .6 \\ .5 & .3 & 1 & .5 & .5 & .4 & .1 & .4 & .7 & .6 \\ .8 & .6 & .5 & 1 & .7 & .5 & .5 & .7 & .5 & .6 \\ .6 & .7 & .5 & .7 & 1 & .6 & .4 & .5 & .8 & .9 \\ .6 & .9 & .4 & .5 & .6 & 1 & .3 & .7 & .7 & .5 \\ .5 & .4 & .1 & .5 & .4 & .3 & 1 & .6 & .2 & .3 \\ .9 & .8 & .4 & .7 & .5 & .7 & .6 & 1 & .4 & .4 \\ .4 & .6 & .7 & .5 & .8 & .7 & .2 & .4 & 1 & .7 \\ .5 & .6 & .6 & .6 & .9 & .5 & .3 & .4 & .7 & 1 \end{bmatrix}$$

By repeating $R^{(n+1)} = R^{(n)} \circ R^{(n)}$ till $R^{(n)} = R^{(n+1)}$.
In this way, similarity-relation $R^{(n)}$ will be calculated as:

$$R^{(n)} = \begin{bmatrix} 1 & .2 & .5 & .8 & .6 & .2 & .3 & .9 & .4 & .3 \\ .2 & 1 & .3 & .6 & .7 & .9 & .2 & .8 & .3 & .2 \\ .5 & .3 & 1 & .5 & .3 & .4 & .1 & .3 & .7 & .6 \\ .8 & .6 & .5 & 1 & .7 & .3 & .5 & .4 & .1 & .3 \\ .6 & .7 & .3 & .7 & 1 & .2 & .4 & .5 & .8 & .9 \\ .2 & .9 & .4 & .3 & .2 & .4 & .1 & .3 & .7 & .2 \\ .3 & .2 & .1 & .5 & .4 & .1 & 1 & .6 & .1 & .3 \\ .9 & .8 & .3 & .4 & .5 & .3 & .6 & 1 & 0 & .2 \\ .4 & .3 & .7 & .1 & .8 & .7 & .1 & 0 & 1 & .1 \\ .3 & .2 & .6 & .3 & .9 & .2 & .3 & .2 & .1 & 1 \end{bmatrix}$$

Now apply [1.] and [2.]

$$\begin{bmatrix} 0 & .7 & 0 & .8 & .6 & .6 & 0 & .9 & 0 & 0 \\ .7 & 0 & 0 & .6 & .7 & .9 & 0 & .8 & .6 & .6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & .7 & .6 \\ .8 & .6 & 0 & 0 & .7 & 0 & 0 & .7 & 0 & .6 \\ .6 & .7 & 0 & .7 & 0 & .6 & 0 & 0 & .8 & .9 \\ .6 & .9 & 0 & 0 & .6 & 0 & 0 & .7 & .7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & .6 & 0 & 0 \\ .9 & .8 & 0 & .7 & 0 & .7 & .6 & 0 & 0 & 0 \\ 0 & .6 & .7 & 0 & .8 & .7 & 0 & 0 & 0 & .7 \\ 0 & .6 & .6 & .6 & .9 & 0 & 0 & 0 & .7 & 0 \end{bmatrix}$$

Firstly, set
$I = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $C = \{\ \}$.
Then apply [3.] and [4.]

3. Now $a_{18} = a_{26} = a_{5\ 10} = 0.9$ are maximum and a_{18} is randomly selected. Then $C = \{1, 8\}$.

4. $a_{12} + a_{82} = a_{14} + a_{84} = 1.5$ are maximum and $j = 4$ is randomly selected. Then $C = \{1, 8, 4\}$.

Repeat [4.]

4. $a_{12} + a_{42} + a_{82} = 2.1$ is maximum, then $C = \{1, 8, 4, 2\}$.

4. There are no j such that $a_{1j} + a_{2j} + a_{4j} + a_{8j}$ is maximum. Then final $C = \{1, 8, 4, 2\}$.

   ⋆ $a_{16} + a_{26} + a_{46} + a_{86} = 0.6 + 0.9 + 0 + 0.7 = 2.2$ seems maximum but actually not because $a_{46} = 0$

   Note that $\sum_{i \in C} a_{iu} = \max_{j \in I \setminus C}\{\sum_{i \in C} a_{ij} | a_{ij} \neq 0\}$

Next

5. Let $I = \{3, 5, 6, 7, 9, 10\}$

3. $a_{5\ 10} = 0.9$ is maximum. Then renew C as $\{5, 10\}$.

4. $a_{59} + a_{10\ 9} = 1.5$ is maximum. Then $C = \{5, 10, 9\}$.

4. There are no j in $\{3, 6, 9\}$ such that $a_{5j} + a_{9j} + a_{10j}$ is maximum. Then final $C = \{5, 10, 9\}$.

Further

5. Let $I = \{3, 6, 7\}$.

3. Now $a_{36} = a_{37} = a_{67} = 0$. Then $\{3\}$, $\{6\}$and $\{7\}$are three separated clusters.

In fact

$$\begin{bmatrix} a_{33} & a_{36} & a_{37} \\ a_{63} & a_{66} & a_{67} \\ a_{73} & a_{76} & a_{77} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

So

$\sum_{i \in \{3,6,7\}} a_{iu} = \max_{j \in \{3,6,7\}} \{\sum_{i \in C} a_{ij} | a_{ij} \neq 0\}$

does not exit any more.

4. The other composition forumula

Besides (i) Max-Min composition:

$$\mu_{R_{A\circ B}}(x,z) = \max\{\min_{y}\{\mu_{R_A}(x,y), \mu_{R_B}(y,z)\}\}$$

we have a cuple of other formulae: (ii) *max-prod*

$$\mu_R(x,y) = \max_{z\in X}\{\mu_R(x,z) \times \mu_R(z,y)\}\}$$

(iii) *max-avg*

$$\mu_R(x,y) = \max_{z\in X}\{(\mu_R(x,z) + \mu_R(z,y))/2\}$$

(iv) *max*-Δ

$$\mu_R(x,y) = \max_{z\in X}\{\max\{0, \mu_R(x,z) + \mu_R(z,y) - 1\}\}$$

E.g.

when

$$\begin{bmatrix} 0.1 & 0.2 & 0.3 \\ 0.4 & 0.5 & 0.6 \\ 0.7 & 0.8 & 0.9 \end{bmatrix} \circ \begin{bmatrix} 0.3 & 0.1 & 0.5 \\ 0.2 & 0.7 & 0.8 \\ 0.9 & 0.4 & 0.0 \end{bmatrix}$$

MAX-PROD

$$\mu_R \circ \mu_R(2,3)$$
$$= \max\{0.4 \times 0.5, 0.5 \times 0.8, 0.6 \times 0.0)\}$$
$$= \max\{0.2, 0.4, 0.0\} = 0.4$$

MAX-AVG

$$\mu_R \circ \mu_R(2,3)$$
$$= \max\{(0.4 + 0.5)/2, (0.5 + 0.8)/2, (0.6 + 0.0)/2\}$$
$$= \max\{0.45, 0.65, 0.3\} = 0.65$$

MAX-Δ

$$\mu_R \circ \mu_R(2,3)$$
$$= \max\{\max(0, 0.4 + 0.5 - 1.0), \max(0, 0.5 + 0.8 - 1.0), \max(0, 0.6 + 0.0 - 1.0)\}$$
$$= \max\{\max(0, -0.1), \max(0, 0.3), \max(0, -0.4)\}$$
$$= \max\{0, 0.3, -0.4\} = 0.3$$

V. Time Series Forecasting by Fuzzy

1. Takagi-Subeno Formula again

R_i: If $y(t-1)$ is A_1^i and $y(t-2)$ is A_2^i and $\cdots$ and $y(t-n+1)$ is A_n^i then $y(t)$ is g^i.

R_i: If $x_1(t)$ is A_1^i and $x_2(t)$ is A_2^i and $\cdots$ and $x_n(t)$ is A_n^i then $y(t)$ is g^i.

2. Fuzzy Time Series

Challenge 1

year	enrolled	actual change	F(t) (degree of membership by vector representation)						predicted change
			big decrease	decrease	no change	increase	big increase	too big increase	
1971	13055								
1972	13563	+508	0	0	0	0.5	1	0.5	
1973	13867	+304	0	0	0	1	0.5	0	
1974	14696	+829	0	0	0	0	0.5	1	
1975	15460	+764							
1976	15311	-149			......	......			
1977	15603	+292							
1978	15861	+258							⋮
1979	16807	+946							
1980	16919	+112							
1981	16388	-531							?
1982	15433	-955							
1983	15497	+64							
1984	15145	-352							
1985	15163	+18							⋮
1986	15984	+821							
1987	16859	+875							
1988	18150	+1291							
1989	18970	+820							
1990	19328	+358							
1991	19337	+9							
1992	18876	-461							

$$
\begin{array}{lccccccc}
 & \text{very very small} & \text{very small} & \text{small} & \cdots\cdots & \text{big} & \text{very big} & \text{too big} \\
F(t) = & [\ 1 \quad 0.5 & 0 & 0 & \cdots\cdots & 0 & 0 & 0\ ]
\end{array}
$$

$$C(t) = F(t-1) = [\, c_1 \quad \cdots\cdots \quad c_m \,]$$

$$O(t) = \begin{bmatrix} F(t-2) \\ \vdots \\ F(t-w-1) \end{bmatrix} \Rightarrow \begin{bmatrix} o_{11} & \cdots\cdots & o_{1m} \\ & \vdots & \\ o_{w1} & \cdots\cdots & o_{wm} \end{bmatrix}$$

$$R(t) = \begin{bmatrix} c_1 \cdot o_{11} & \cdots\cdots & c_m \cdot o_{1m} \\ & \vdots & \\ c_w \cdot o_{w1} & \cdots\cdots & c_w \cdot o_{wm} \end{bmatrix} \Rightarrow \begin{bmatrix} R_{11} & & R_{1m} \\ & \vdots & \\ R_{w1} & \cdots\cdots & R_{wm} \end{bmatrix}$$

$$F(t) = [\ \max(R_{11}, R_{21}, \cdots, R_{w1}) \quad \cdots\cdots \quad \max(R_{1w}, R_{22}, \cdots, R_{ww})\ ]$$

Apply the method in the previous page

1. *With m being 6, i.e., big decrease, decrease, no change, increase, big increase, too big increase, estimate $F(1977)$ by the data from 1972 to 1976 ($w = 6$)*
2. *calculate the center of gravity. This is the predicted value of the year*
3. *Then predict 1978 in the same way*
4. *Repeat 2. and 3. till 1992*
5. *Plot the points predicted, and compare the actual data*

Yet another dataset

Date	Open	Close	Date	Open	Close
9/26/2007	13779.3	13878.15	8/9/2007	13652.33	13270.68
9/25/2007	13757.84	13778.65	8/8/2007	13497.23	13657.86
9/24/2007	13821.57	13759.06	8/7/2007	13467.72	13504.3
9/21/2007	13768.33	13820.19	8/6/2007	13183.13	13468.78
9/20/2007	13813.52	13766.7	8/3/2007	13462.25	13181.91
9/19/2007	13740.61	13815.56	8/2/2007	13357.82	13463.33
9/18/2007	13403.18	13739.39	8/1/2007	13211.09	13362.37
9/17/2007	13441.95	13403.42	7/31/2007	13360.66	13211.99
9/14/2007	13421.39	13442.52	7/30/2007	13266.21	13358.31
9/13/2007	13292.38	13424.88	7/27/2007	13472.68	13265.47
9/12/2007	13298.31	13291.65	7/26/2007	13783.12	13473.57
9/11/2007	13129.4	13308.39	7/25/2007	13821.4	13785.79
9/10/2007	13116.39	13127.85	7/24/2007	13940.9	13716.95
9/7/2007	13360.74	13113.38	7/23/2007	13851.73	13943.42
9/6/2007	13306.44	13363.35	7/20/2007	14000.73	13851.08
9/5/2007	13442.85	13305.47	7/19/2007	13918.79	14000.41

9/4/2007	13358.39	13448.86	7/18/2007	13955.05	13918.22
8/31/2007	13240.84	13357.74	7/17/2007	13951.96	13971.55
8/30/2007	13287.91	13238.73	7/16/2007	13907.09	13950.98
8/29/2007	13043.07	13289.29	7/13/2007	13859.86	13907.25
8/28/2007	13318.43	13041.85	7/12/2007	13579.33	13861.73
8/27/2007	13377.16	13322.13	7/11/2007	13500.4	13577.87
8/24/2007	13231.78	13378.87	7/10/2007	13648.59	13501.7
8/23/2007	13237.27	13235.88	7/9/2007	13612.66	13649.97
8/22/2007	13088.26	13236.13	7/6/2007	13559.01	13611.68
8/21/2007	13120.05	13090.86	7/5/2007	13576.24	13565.84
8/20/2007	13078.51	13121.35	7/3/2007	13556.87	13577.3
8/17/2007	12848.05	13079.08	7/2/2007	13409.6	13535.43
8/16/2007	12859.52	12845.78	6/29/2007	13422.61	13408.62
8/15/2007	13021.93	12861.47	6/28/2007	13427.48	13422.28
8/14/2007	13235.72	13028.92	6/27/2007	13336.93	13427.73
8/13/2007	13238.24	13236.53	6/26/2007	13352.37	13337.66

Challenge 2

Year to Year	Change
1971-1972	3.89%
1972-1973	2.24%
1973-1974	5.98%
1974-1975	5.20%
1975-1976	-0.96%
1976-1977	1.91%
1977-1978	1.65%
1978-1979	5.96%
1979-1980	0.67%
1980-1981	-3.14%
1981-1982	-5.83%
1982-1983	0.41%
1983-1984	-2.27%
1984-1985	0.12%
1985-1986	5.41%
1986-1987	5.47%
1987-1988	7.66%
1988-1989	4.52%
1989-1990	1.89%
1990-1991	0.05%
1991-1992	-2.38%

Year	Actual Enrollment	Actual %	Fuzzy Set	t_j Predicted %	Forcast
1971	13055				
1972	13563	3.89	X_9	2.7229	13410
1973	13867	2.24			
1974	14696	5.98			
1975	15460	5.20			
1976	15311	-0.96			
1977	15603	1.91		⋮	
1978	15861	1.65			
1979	16807	5.96			
1980	16916	0.67			
1981	16388	-3.14			
1982	15433	-5.38		?	
1983	15497	0.41			
1984	15145	-2.27			
1985	15163	0.12			
1986	15984	5.41		⋮	
1987	16859	5.47			
1988	18150	7.66			
1989	18970	4.52			
1990	19328	1.89			
1991	19337	0.05			
1992	18876	-2.38			

Fuzzy Set :

X_1 = very very small (-6.0, -4.0)

⋮

X_{13} = very too large (6.0, 8.0)

a_j

t_j

Intervals	Number of Data
-6.0, -4.0	1
-4.0, -2.0	1
-2.0, 0.0	2
0.0, 2.0	7
2.0, 4.0	3
4.0, 6.0	6
6.0, 8.0	1

→

Linguistic	Intervals
X 1	-6.0, -4.0
X 2	-4.0, -2.0
X 3	-2.0, 0.0
X 4	0.0, 0.5
X 5	0.5, 1.0
X 6	1.0, 1.5
X 7	1.5, 2.0
X 8	2.0, 3.0
X 9	3.0, 4.0
X 10	4.0, 4.7
X 11	4.7, 5.3
X 12	5.3, 6.0
X 13	6.0, 8.0

Devide the interval with the largest number of data into 4 sub-interval of equal length 2nd largest into 3, and 3rd largest into 2 with all other intervals remain unchanged.

Defuzzification

$$t_j = \begin{cases} \dfrac{1.5}{\frac{1}{a_1}+\frac{1}{a_2}} \ \ldots \ \text{if } j = 1 \\[2ex] \dfrac{2}{\frac{0.5}{a_{j-1}}+\frac{1}{a_j}+\frac{1}{a_{j+1}}} \ \ldots \ \text{if } 2 \le j \le (n-1) \\[2ex] \dfrac{1.5}{\frac{0.5}{a_{n-1}}+\frac{1}{a_n}} \ \ldots \ \text{if } j = n \end{cases}$$

- where a_{j-1}, a_j, a_{j+1} are the midpoints of the fuzzy intervals X_{j-1}, X_j, X_{j+1} respectively.

- t_j yields the predicted year to year percentage change of enrollment.

- Use the predicted percentage on the previous years enrollment to determine the forecasted enrollment.

Yet another dataset

Date	Open	Close	Date	Open	Close
9/26/2007	13779.3	13878.15	8/9/2007	13652.33	13270.68
9/25/2007	13757.84	13778.65	8/8/2007	13497.23	13657.86
9/24/2007	13821.57	13759.06	8/7/2007	13467.72	13504.3
9/21/2007	13768.33	13820.19	8/6/2007	13183.13	13468.78
9/20/2007	13813.52	13766.7	8/3/2007	13462.25	13181.91
9/19/2007	13740.61	13815.56	8/2/2007	13357.82	13463.33
9/18/2007	13403.18	13739.39	8/1/2007	13211.09	13362.37
9/17/2007	13441.95	13403.42	7/31/2007	13360.66	13211.99
9/14/2007	13421.39	13442.52	7/30/2007	13266.21	13358.31
9/13/2007	13292.38	13424.88	7/27/2007	13472.68	13265.47
9/12/2007	13298.31	13291.65	7/26/2007	13783.12	13473.57
9/11/2007	13129.4	13308.39	7/25/2007	13821.4	13785.79
9/10/2007	13116.39	13127.85	7/24/2007	13940.9	13716.95
9/7/2007	13360.74	13113.38	7/23/2007	13851.73	13943.42
9/6/2007	13306.44	13363.35	7/20/2007	14000.73	13851.08
9/5/2007	13442.85	13305.47	7/19/2007	13918.79	14000.41
9/4/2007	13358.39	13448.86	7/18/2007	13955.05	13918.22
8/31/2007	13240.84	13357.74	7/17/2007	13951.96	13971.55
8/30/2007	13287.91	13238.73	7/16/2007	13907.09	13950.98
8/29/2007	13043.07	13289.29	7/13/2007	13859.86	13907.25
8/28/2007	13318.43	13041.85	7/12/2007	13579.33	13861.73
8/27/2007	13377.16	13322.13	7/11/2007	13500.4	13577.87
8/24/2007	13231.78	13378.87	7/10/2007	13648.59	13501.7
8/23/2007	13237.27	13235.88	7/9/2007	13612.66	13649.97
8/22/2007	13088.26	13236.13	7/6/2007	13559.01	13611.68
8/21/2007	13120.05	13090.86	7/5/2007	13576.24	13565.84
8/20/2007	13078.51	13121.35	7/3/2007	13556.87	13577.3
8/17/2007	12848.05	13079.08	7/2/2007	13409.6	13535.43
8/16/2007	12859.52	12845.78	6/29/2007	13422.61	13408.62
8/15/2007	13021.93	12861.47	6/28/2007	13427.48	13422.28
8/14/2007	13235.72	13028.92	6/27/2007	13336.93	13427.73
8/13/2007	13238.24	13236.53	6/26/2007	13352.37	13337.66