(Brest State Technical University 2005 Spring Semester: Corse Practice)

Associative Memory by Hopfield Neural Network

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1 1st week

The first task of this practice is to create your own hopfield network as follows, today.

- Number of neuron $N = 100$
- At time t, neuron j takes a state $S_i(t) = 1$ or -1 $\quad (i = 0, 1, 2, \cdots, 99)$.
  - Time here is descrete, that is, $(t = 0, 1, 2, \cdots)$.
- All the neuron inclucing itself connectted with each other.
  - Hence total connections are $N^2 = 10,000$
- Weight value from neuron i to j denoted as w_{ji} takes a random value from -1.0 to 1.0, today.

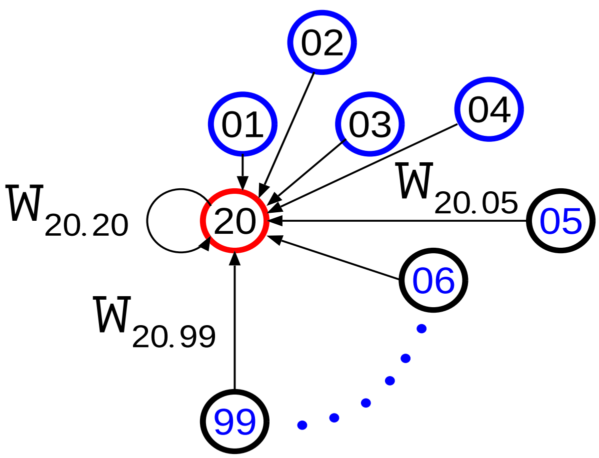

Figure 1:

The state of neuron at time $t+1$ will be ON $(+1)$ if all the weighted sum from all neurons (the other 99 neurons and itself) is positive, otherwise OFF (-1). That is,

$$S_i(t+1) = \mathrm{sgn}(\sum_{j=0}^{99} w_{ij} S_i(t)) \tag{1}$$

Excersize 1 *On your monitor screen, display 100 neuron each of which takes a state of -1 or 1 at random. This is the pattern at $t = 0$. Then renew states from one time to the next and display states of 100 neurons.*

2 2nd week

In the hopfield network created in the 1st week, we try to store a pttern. In the 1st week both state configuration and weight values are random, but this week we specify them.

The pattern is constructed as 2-dimensional array of $N = 100$ neurons like below.

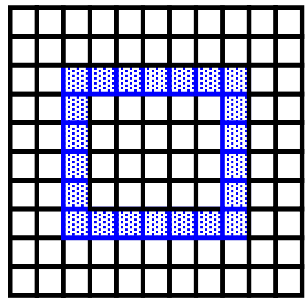

Figure 2:

We interpret it as 2-dimensional pixels, not color, not gray-scale, but black-and-white ones. That is, The state of 1 is black and the state of -1 is white. Hence, today $w_{ij} = w_{ji}$.

Weight value is determined such that if and only if the both ends of the connection is 1, the weight value of the connection is 1, otherwise -1. The weight value of the conection from one neuron to itself are set to 0, that is, $w_{ii} = 0$, today. That is,

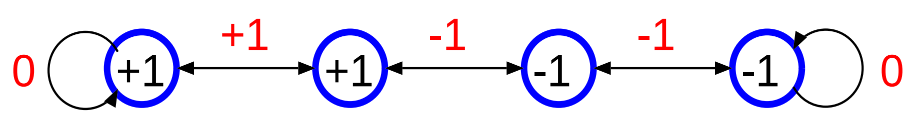

Figure 3: An example of specifying weight values. The value is $+1$, -1 or 0 depending on states of neurons of the both end of the connection.

Or, if you like mathematics, the fomula of determining w_{ij} is

$$w_{ij} = \xi_i \xi_j \quad (i \neq j), \quad w_{ii} = 0 \tag{2}$$

where ξ_i is the state of i-th neuron.

Excersize 2 *On your monitor screen, display 100 neuron each of which takes a state of* -1 *or 1 such that those neurons whose state 1 construct a border of rectangle while neurons of* -1 *are backgraound. Specify weight of connections using Eq. (4). Start with some noisy pattern try to observe at time is* $1, 2, \cdots$.

3 3rd week

With the same frame work as the 2nd week, let's increase the number of patterns. After storing the 1st pattern as you did last time, add the next patterns one by one adding +1 to the connection weight if both end of the connection is +1 for the 2nd pattern otherwise adding -1. To be more specific:

$$w_{ij} = \sum_{\mu=1}^{p} \xi_i^\mu \xi_j^\mu \quad (i \neq j), \quad w_{ii} = 0. \tag{3}$$

As a result if you store p patterns the weight values ranges from $-n$ to n.

Excersize 3 *Try to store multiple patterns and observe how it is clever. As an experiment, try to start with the pattern of all white pixcels.*

4 4th week

Then we are curious how many patterns we can store.

Excersize 4 *Try to store as many as patterns so that we observe a confusion.*

5 An additional week – For those who failed

Hopfield network can also store animation made up of p patterns, by specifying weights as:

$$w_{ij} = \sum_{\mu=1}^{p} \xi_i^{\mu+1} \xi_j^\mu \quad (i \neq j), \quad w_{ii} = 0. \tag{4}$$

Excersize 5 *Prepare, say, 3 patterns and specify weights with the above equation. Observe the result starting with one of those 3 patterns.*