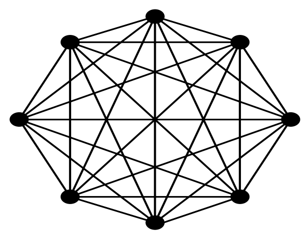
Hopfield Network — Associative memory

Akira Imada

1 Hopfield Neural Network

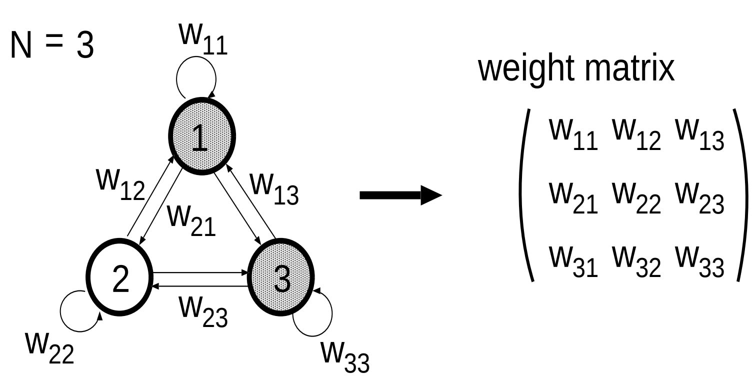
1.1 Hopfield NN is a Fully-connected Neural Network like



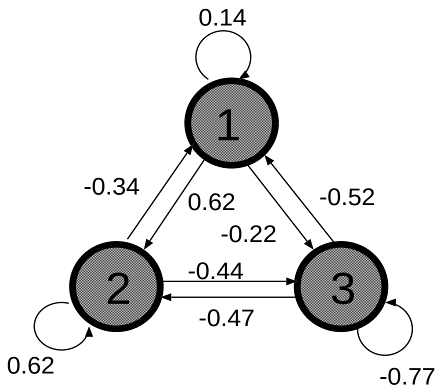
1.2 State Transition

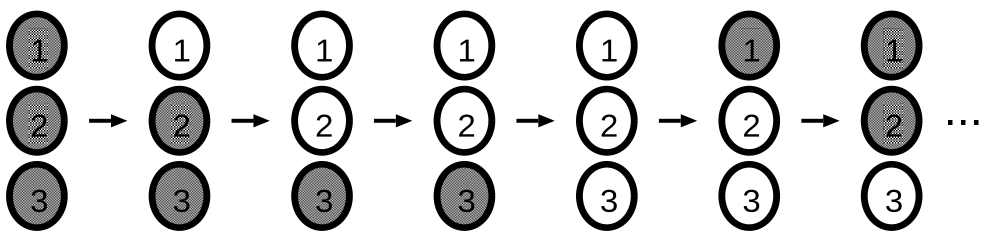
$$s_i(t+1) = sgn\left(\sum_{j\neq i}^{N} w_{ij} \cdot s_j(t)\right). \tag{1}$$

1.3 A Toy Model with Three Neurons



1.4 Examples of State Trajectory:(Chaotic Trajectry)





An example of state transition in the above toy example:

$$\begin{aligned} s_1(1) &= sgn\ (\ w_{11} \cdot s_1(0) + w_{12} \cdot s_2(0) + w_{13} \cdot s_3(0)\ ) \\ &= sgn\ (\ 0.14 \cdot 1 + (-0.34) \cdot 1 + (-0.52) \cdot 1\ ) \\ &= -1 \end{aligned}$$

Excersise 1 *Simulate Hopfield Network with N bipolar neurons (i.e., $S_i(t) = 1\, or - 1$) whose states are updated with Eq. (1). Give a real random value ($\in \{-1, 1\}$) to each of the w_{ij}. and display all the neuron's state in a line (one-dimensional) of the screen of your PC. These are initial state of N neurons ($t = 0$). You might use other symbol like • and ∘ instead of "1" and " −1", respectively.*

Then update those state one time to the next acording to Eq. (1) taking a sleep-time, say, for 1 second between time t and $(t + 1)$.

1.5 Hebbian Weights

To store p patterns of N-bits of 1 and -1 (bipolar pattern)

$$(\xi_1^1, \xi_2^1, \cdots, \xi_N^1), \ \ (\xi_1^2, \xi_2^2, \cdots, \xi_N^2), \ \ (\xi_1^3, \xi_2^3, \cdots, \xi_N^3), \ \ \cdots, \ \ (\xi_1^p, \xi_2^p, \cdots, \xi_N^p). \tag{2}$$

We may specify w_{ij} using so called *Hebbian Learning*

$$w_{ij} = \frac{1}{N} \sum_{\nu=1}^{p} \xi_i^\nu \xi_j^\nu \ \ (i \neq j), \ \ w_{ii} = 0. \tag{3}$$

- For example, to store only $(1, 1, 1)$ in our toy-network with 3 neurons.[1]

$$w_{ij} = \begin{pmatrix} 0 & 0.11 & 0.11 \\ 0.11 & 0 & 0.11 \\ 0.11 & 0.11 & 0 \end{pmatrix}$$

Excersise 2 *This time simulatin is with N^2 bipolar neurons. The states of N neurons are displayed from one line to the next with N lines in total. Namely, like 2-dimensional patterns with $N \times N$ piccels. Then update those state as before acording to Eq. (3) but a sleep-time of ,e.g., 1 second from time t to $(t+1)$ is between two patterns. Then try an experimentthe folloing the next procedures.*

1 *Create your own p patterns with $N \times N$ 1 and -1. using $\bullet$ and $\circ$ as before.*

- *For example, with $p = 5$ and $N = 10$.*
- *Patterns could be a series of national flags, faces, houses, cars. But Simple patterns like stripes or cross are easy to be treated.*

2 *Show the animation by updating states with Eq.(**??**) as before, starting with one of your patterns.*

- *For example, with $p = 5$ and $N = 10$.*

3 *Calculate w_{ij} using Eq.(3). Then under this w_{ij} starting with one of your patterns, update the state as 2.*

4 *Give nises to one of your patterns by flipping the biths (1 $\rightarrow -1$ and vice versa), then try 3. starting with this noisy pattern.*

- *Try experiments with the rate of noises with* $10\%, 20\%, \cdots, 60\%$

[1] Many other solutions of a *weight configuration* to store a same set of patterns exist, and each of these solutions can *recall* stored pattern from its partial input; and/or incorrect input but has a different *basin of attraction*; *storage capacity*.

1.6 Analysis to Report

Excersise 3 *The performance depends on (1)the number of patterns to be stored; (2) how close each patterns is with each other? (3) rate of noises; and so on. Then try experiments by changing these three conditions.*

- *Report your results of analysis like as follows:*
    - *(1) Show all of your patterns you stored.*
    - *(2) If recovery from 50% of input noises is necessary, how many patterns can be stored? (They say about 13% of number of neurons, theoreticaly.)*
    - *(3) Under this upper limit storage, show a series of snapshots of the pattern starting from 50% noisy input of one of the stored patterns until this input converges to the perfect (original) pattern.*

(An example of how you create your report)

An Experiment using the Hopfield Neural Network

Ivan Ivanovich Ivanov

Abstract

Here you summarize briefly what you are going to write in this paper. That is (i) What are you doing? (ii) Why are you doing? (iii) Which theory/equation/metods are you using in this experiment? (iV) What you are expecting to observe. (v) etc...

1. INTRODUCTION

Here you describe "What are of your interest?" "Why it is important?" "What are you expecting?" etc... those you wrote in *"abstract"* above, more in detail. Like, We are interesting a model of human memory (Hopfield, 1982) [2] .

2. METHOD

Here you describe "What did you use?" That is, Theory, Equation, or whatever. For example, we are going to proof the equation so-called "The most bearutiful formula"

$$e^{\pi} = 1$$

If you should use the method that you have found in other paper, you should also refer it as follows. We use the method proposed by (Imada, 1999).

3. EXPERIMENT

Here you expalin "What you did?" For example, you may start with a series of patterns you stored in your Hopfield network, like

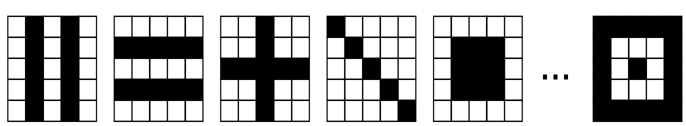


Figure 1: The patterns we store in our Hopfield Networks with 25^2 neurons.

4. RESULTS & DISCUSSIONS

You show your results here. The results should look *impressive*. Using Figures are highly recommended, like

[2] If you cite some paper you should refer to it in the final section named *Reference*.

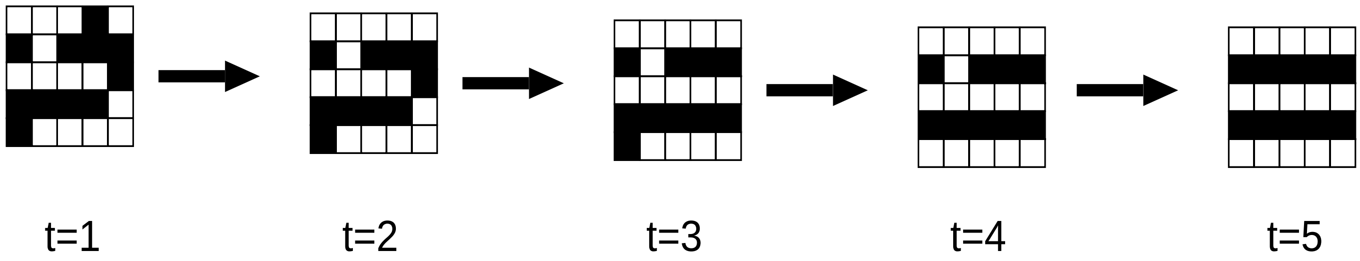


Figure 2: A series of snapshots along the trajectry from noisy input to convergence, starting with 20% noisy input untill it converges to the original pattern perfectly.

5. CONCLUSION

Here you summarise what you conclude from your observations.

REFERENCE

Hopfield, J. J. (1982) *"Neural Networks and Physical Systems with Emergent Collective Computational Abilities."* In Proceedings of the National Academy of Sciences, USA 79, pp.2554-2558.

Imada, A. (1999) *"What does a Peak in the Landscape of a Hopfield Associative Memory Look Like?"* In Proceedings of International Work-Conference on Artifcial and Natural Neural Networks, Vol. 1, Springer Verlag, Lecture Notes in Computer Science, No.1606, pp.357–366.